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SINGLE PARAMETER TESTING

Phase A - Completion Report

12 November 1964

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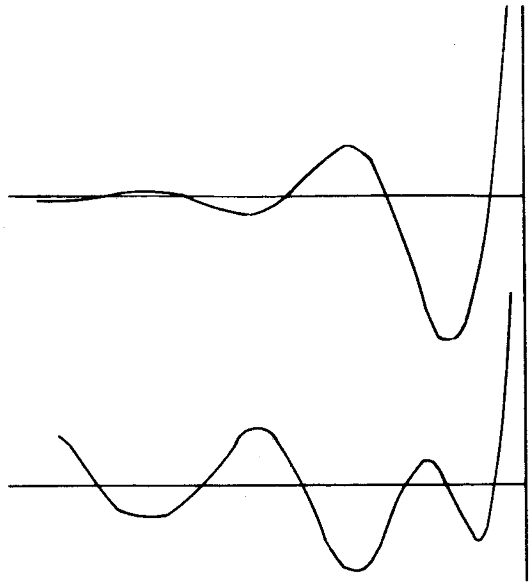


S I N G L E   P A R A M E T E R   T E S T I N G

"There is a better way to conduct testing"

SUMMARY: This report gives the first phase results obtained on NAS8-11715 contract, in the area of single parameter testing. The main objective of the study is to put into operation better ways of testing transfer functions. The expected savings are faster checkout time, better accuracy, and less degradation of performance due to the testing.

The result of the first phase study is - YES! We can test linear passive first and second order transfer functions. The savings are faster checkout time, faster isolation of failures, and less degradation of performance due to testing.

The technical areas investigated have resulted in two techniques, which are directly applicable to the solution of single parameter testing of linear passive networks. These techniques are:

1. Growing exponentials as a probing signal for parameter testing.
2. Optimization of feedback control for system testing.

Each of the preceding areas have their advantages. Growing exponentials have the advantage of being able to measure many parameters (4 have been measured successfully, an upper limit has not been established) with one probing signal. Feedback control has the advantage of possibly less testing equipment, with measurement of the combined effects of many parameters (2 parameter effects have been observed, an upper limit has not been established).

What areas require more investigation? The growing exponential technique needs to be extended to third order systems.

The optimization feedback technique needs to be investigated in more detail, so as to establish its limitations.

The area of testing active networks needs to be investigated, so as to provide techniques in this area.

Where are we going? The next phase will be to investigate the areas of active networks. This area was chosen so that in the third phase of the study, practical application of testing a particular real system will be accomplished. The other established applicable techniques which will be extended within the time and money available.

SECTION 1

INTRODUCTION

The single parameter testing program was established to perform mathematical analysis on typical systems of varying complexity. The program will verify the applicability of single parameter testing to launch vehicle system checkout.

The general technical approach of the study was limited to systems or devices for which continuous transfer functions can be written and restricted to their linear regions. Primary emphasis was directed on the identification of changes in the terms which compose the transfer function.

The approach used was to:

1. Describe transfer functions and study the changes in behavior with incremental changes in its parameter, i.e., terms.
2. For each transfer function, investigate the measurability of performance degradation due to changes in the transfer function. One output of this task will be determining the feasibility of GO, NO-GO, decisions based on parameter testing.
3. Investigate possible theories of measuring single parameters to accomplish the measurement of incremental changes, and performance degradation due to transfer function changes.

The study is divided into three specific tasks:

Phase A: Simple first, second, and third order linear passive networks whose transfer functions resemble those of useful systems, were to be selected for detailed investigation. The results will be used to extend the testing to higher order systems.

Phase B: The investigation and selection of criteria described in Phase A. This is to include the linear active networks.

Phase C: With the guidance and approval of the NASA technical representative, an actual subsystem will be chosen for analysis. The transfer function will be derived, and the techniques developed in Phase A and B are to be applied to the subsystem.

SECTION 2

THEORIES INVESTIGATED

The application of various theories were investigated. These areas are listed below with reason for discarding or further investigation.

THEORY	CONCLUSION
a. Growing exponentials as a probing signal for single parameter testing.	A detailed analysis has led to the measurement of first and second order systems. The results are favorable, and imply the testing of many parameters with one probing signal, in a short period of time.
b. Optimization of feedback control for system testing.	A preliminary analysis on a first order system had favorable results. More investigation is necessary to establish the limits of this technique.
c. Impulse testing.	Impulse testing was reviewed and discarded for two reasons: 1. The application of impulse testing is limited to low frequency systems, because the sampling of impulse reponses by computers is limited by the sampling frequencies. 2. Impulse testing has a degradation effect upon the equipment being tested, because of the "shock" imposed by the impulse.
d. Correlation testing.	Correlation testing has the same disadvantages as impulse testing. 1. The application is limited to low frequency systems, which can be sampled by computers. 2. The time required for correlation testing is excessive, a correlation test requires the time average of two signals for each data point.

		At least 10 or more data points would be necessary for measuring the correlation function.
e.	Network synthesis using sinusoidal measurements.	These types of measurements are characterized by extended periods of time. The sinusoidal signals must be varied over a variety of frequencies to synthesize the network.
f.	State variable estimation (parameters are considered to be a state variable and the state is estimated by statistical synthesis of noise outputs).[16]	This technique requires an extensive amount of sample data, and analysis time by a computer. The relationships between the states are estimated, and may have statistical variations.

SECTION 3

GROWING EXPONENTIALS

3.1 INTRODUCTION

Growing exponentials appear to have the best liklihood of successfully solving the problem of single parameter testing. In the following pages, the theory will be presented which allows testing by growing exponentials.

The use of growing exponentials has been investigated by Huggins, et al, in such applications as electrocardiography and in identification of static nonlinear operators, and in system identification problems.[1-15]

This method was actively studied in detail with the objective of applying it generally to transfer functions. The general instrumentation scheme for measurement is shown in Figure 3-1.

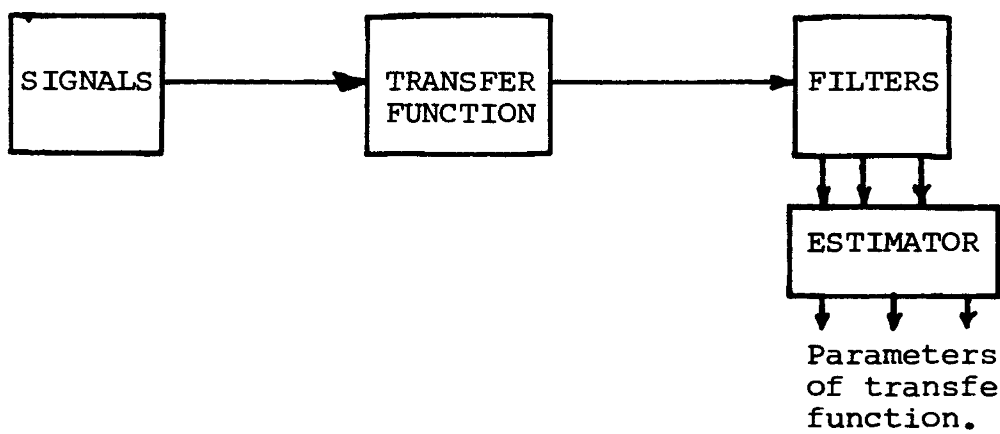
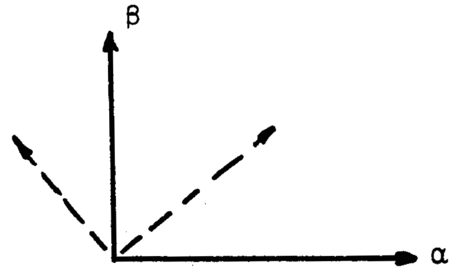


Figure 3-1
Instrumentation Scheme

In a generalized point of view, the signals are a collection of vectors in "n" dimensional space. Each of these vectors are orthogonal, so that each are independent.

If the transfer function can be expanded in terms of orthogonal linear stationary operators, then the measurement of these operators can be accomplished by measuring the projections of the signal vectors on to the linear operator space.

For example, assume that the operator space is as illustrated in Figure 3-2



Orthogonalized Portions of Some Transfer Functions

Figure 3-2

where α and β are two orthogonalized portions of some transfer function. Also assume that the signals are <u>any</u> orthogonal set as illustrated by the dashed lines in Figure 3-2. The projections of these signal vectors on the operator space will, when multiplied by scale factors, measure the magnitude of α and β. Normally α and β will be proportional to a change in some parameter.

3.2 ORTHOGONALIZED SIGNALS

To obtain orthogonalized signals, it is sufficient to have the time average of the interproduct of the signals zero. Let $f_i(t)$ be the i^{th} signals, then

$$\int_{-\infty}^{+\infty} f_i^*(t)\, f_j(t)\, dt = 0 \qquad (3\text{-}1)$$

when $i \neq j$, otherwise

$$\int_{-\infty}^{+\infty} f_i^*(t)\, f_i(t)\, dt = 1 \qquad (3\text{-}2)$$

Now in order to obtain orthogonal signals, exponentials may be considered. Exponentials have several advantages over other sets of orthogonal functions. These include relatively short time bases, and capabilities of being matched to the system to be tested.

Orthogonalization of the exponentials may be accomplished by the Kautz method.[10] This method allows the approximation of the impulse response of any network by sums of orthogonalized signals. These signals, for a transfer function with all real poles and a higher order denominator than numerator, become:

$$\Phi_n(S) = \sqrt{-S_n - \overline{S}_n} \qquad \frac{(S + \overline{S}_1) \cdot \cdot \cdot \cdot (S + \overline{S}_{n-1})}{(S - S_1) \cdot \cdot \cdot \cdot (S - S_n)} \qquad (3\text{-}3)$$

where S_n = complex frequency with a <u>negative</u> real part of the $n^{\underline{th}}$ exponential component

$\overline{S}_n$ = conjugate of S_n

For example, let the transfer function be

$$H(S) = \frac{1}{S + K_1} \qquad (3\text{-}4)$$

Then the set of orthogonalized exponentials are:

$$\Phi_1(S) = \sqrt{2K_1} \quad \frac{1}{S + K_1} \tag{3-5}$$

$$\Phi_2(S) = \sqrt{2K_1} \quad \frac{(S - K_1)}{(S + K_1)^2} \tag{3-6}$$

$$\Phi_3(S) = \sqrt{2K_1} \quad \frac{(S - K_1)^2}{(S + K_1)^3} \tag{3-7}$$

$$\vdots$$

In the time domain this set of signals are:

$$f_i(t) = \sqrt{2K_1} \quad e^{-K_1 t} \tag{3-8}$$

$$f_2(t) = \sqrt{2K_1} \quad (1 + 2\ t)\ e^{-K_1 t} \tag{3-9}$$

$$\vdots$$

In the following text we will be concerned with <u>negative</u> time functions and sampling at time zero. When considering negative time, the set of orthogonalized components become

$$\Phi_1(S) = \frac{\sqrt{2K_1}}{-S + K_1} \tag{3-10}$$

$$\Phi_2(S) = \frac{\sqrt{2K}}{(-S + K_1)^2} \tag{3-11}$$

$$\vdots$$

And in the time domain

$$f_1(t) = \sqrt{2K_1}\, e^{K_1 t} \quad \text{for } t < 0 \tag{3-12}$$

$$f_2(t) = \sqrt{2K_1}\, e^{K_1 t} \quad (1 + 2\, t) \quad \text{for } t < 0 \tag{3-13}$$

$$\vdots$$

When the transfer function has complex poles, then the orthogonalization takes the form of[10]

$$\frac{\Phi_{2v-1}(S)}{\Phi_{2v}(S)} = \sqrt{2\alpha_v}\; \frac{\left[(S - \alpha_1)^2 + \beta_1^{\,2}\right]\ldots\left[(S - \alpha_{v-1})^2 + \beta_{v-1}^{\,2}\right] (S \pm |S_v|)}{\left[(S + \alpha_1)^2 + \beta_1^{\,2}\right]\ldots\left[(S + \alpha_{v-1})^2 + \beta^2_{v-1}\right]\left[(S+\alpha_v)^2 + \beta_v^{\,2}\right]} \tag{3-14}$$

where $v = 1, 2 - - - n/2$

and the poles are at

$$S_v = -\alpha_v - j\beta_v \quad \text{and} \quad \bar{S}_v = -\alpha_v + j\beta_v. \tag{3-15}$$

The upper (plus) sign pertains to Φ_{2v-1} and the lower sign pertains to $\Phi_{2v}(S)$.

3.3 ORTHOGONAL SEPARATION OF THE SIGNALS

To separate these orthogonalized signals, we only need to accomplish the integral

$$\int_{-\infty}^{+\infty} f_i^*(t)\, f_j(t)\, dt = \begin{cases} 1 & i = j \\ 0 & i \neq j \end{cases} \tag{3-16}$$

This can be performed by performing the contour integration in the frequency domain, i.e., Parseval's Theorem for aperiodic functions.[20]

$$\int_{-\infty}^{+\infty} f_i(t)\, f_j(t) = \int_c \Phi_i^*(-S)\, \Phi_j(S)\, \frac{dS}{2\pi j} \qquad (3\text{-}17)$$

Note that $\Phi_i^*(-S)$ is a real filter which has an impulse response $f_i(t)$ in positive time. The integration in the complex plane is equivalent to sampling the results at time $t = 0$.

3.4 ORTHOGONALIZED TRANSFER FUNCTION

Consider a system H(S) as a function of its parameter variations around some specified nominal design values. Then a Taylor's Series Expansion can be written as:

$$H(S) = H_o + \frac{\partial H(S)}{\partial \alpha_1}\alpha_1 + \frac{\partial H(S)}{\partial \alpha_2}\alpha_2 + \ldots \tfrac{1}{2}\frac{\partial^2 H}{\partial^2 \alpha_1}\alpha_1^2 + \ldots. \qquad (3\text{-}18)$$

where H_o = the specified nominal system and

$\frac{\partial H(S)}{\partial \alpha_i} = H_i(S)$ = first partial derivative of the system with respect to the i^{th} parameter.

Thus, for small deviations in the parameters, the actual system may be decomposed into the sum of the partial systems, $H_i(S)$.

Now, let any transfer function be

$$H(S) = \frac{N(S)}{D(S)} = \frac{\sum_{n=0}^{N} c_n S_n}{\sum_{n=0}^{N} d_n S^n} \quad \text{where } n = 0, 1, 2, 3 \text{ and } c_o = 1, \text{ or } d_o = 1 \qquad (3\text{-}19)$$

To test and determine the transfer function, first the N(S) and then D(S) is tested by proper control of input signals.

H(S) can be expanded in terms of the Taylor Series Expansion for small variations in c_n and d_n as

$$H(S) = H_o(S) + \sum_{n=0}^{n} \frac{\partial H(S)}{\partial c_n} \Delta c_n + \sum_{n=0}^{n} \frac{\partial H}{\partial d_n} \Delta d_n \tag{3-20}$$

Since H(S) is the summation of the changes in c_n and d_n, we can measure first, the Δc_n and then, the Δd_n and combine the answers to obtain $\Delta H(S)$.

Note also that

$$\sum_{n=0}^{n} \frac{\partial H(S)}{\partial c_n} = \sum_{n=0}^{n} \frac{S^n}{D(S)} \tag{3-21}$$

$$\sum_{n=0}^{n} \frac{\partial H(S)}{\partial d_n} = \sum_{n=0}^{N} \frac{-S^n N(S)}{[D(S)]^2} = -\sum_{n=0}^{N} \frac{\partial H}{\partial c_n} H(S) \tag{3-22}$$

Any given $\frac{\partial H(S)}{\partial c_i}$ is orthogonal to any other partial derivative $\frac{\partial H(S)}{\partial c_j}$.

Likewise, any given $\frac{\partial H(S)}{\partial d_i}$ is orthogonal to any other partial derivative $\frac{\partial H(S)}{\partial d_j}$.

This property allows independent measurement of the relative magnitude of all the c_i's or d_i's when only the c_i's or d_i's are measured. The absolute magnitude can be determined by noting the ratio of $\frac{c_o}{d_o}$.

3.5 PARAMETER EFFECTS UPON THE MEASURED SIGNALS

Each of the testing signals, when passed through the system, will be filtered by each partial system. The outputs of these particular systems are combined and then passed through the output filters. The process is illustrated in Figure 3-3.

The mathematical description of passing the inputs through the particular system, filter and estimator will now be given in detail. This analysis will be general and apply to $n^{\underline{th}}$ order transfer functions.

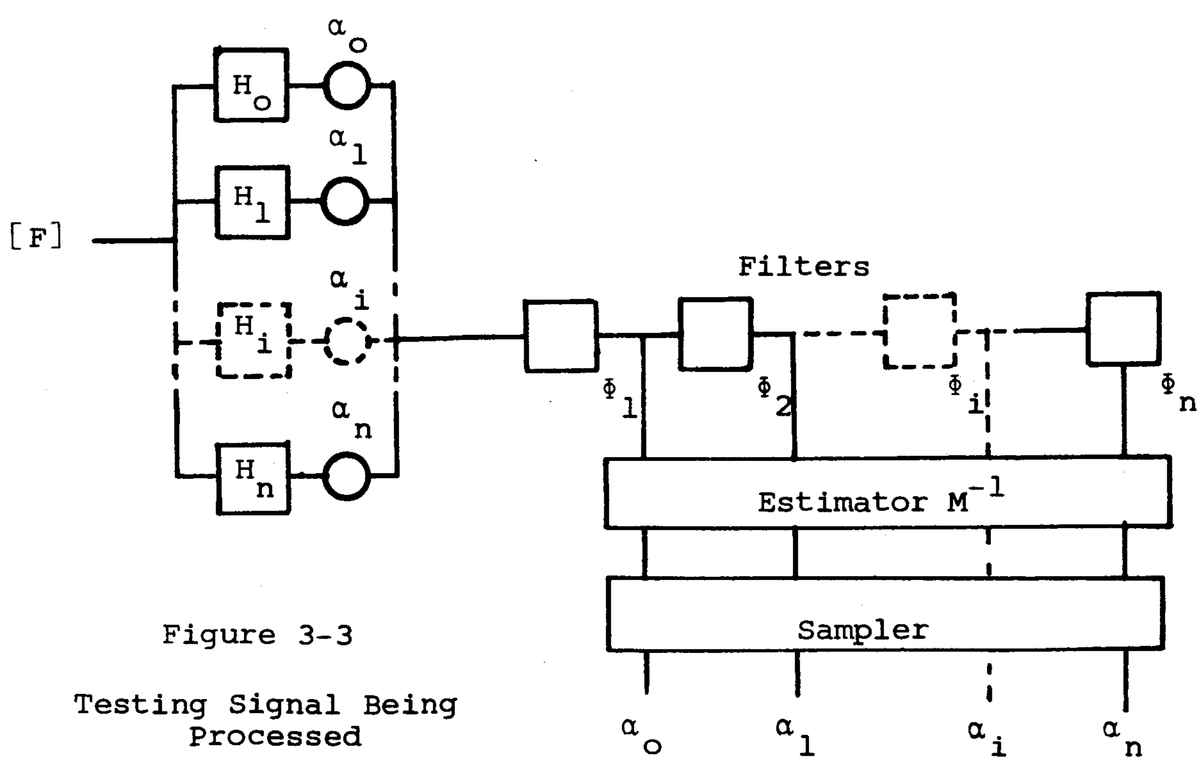


Figure 3-3

Testing Signal Being Processed

A representative of the signals appearing at the output of the $H_i(S)$ component system is

$$H_i(S) \quad [F] \tag{3-23}$$

where [F] is a row matrix of the input probing signals. The output at the first filter is

$$\Phi_1^*(-S) \quad H_i(S) \quad [F] \tag{3-24}$$

Likewise, the out at the $j^{\underline{th}}$ filter is

$$H_j = \Phi_j^*(-S) \quad H_i(S)\,[F], \tag{3-25}$$

which is a row matrix denoted H_j.

The collection of these row matrices denotes a modulation matrix, H_α. H_α describes the effects of $H_i(S)$ on the outputs of the filters under the influence of the input probing signals. The object will be to sample outputs of these filters at a particular time. These samples will represent the variation in the parameters tested.

The output of $j^{\underline{th}}$ filter can be obtained at any particular time by performing the following integration in the complex plane:

$$h_{jk}(\tau) = \int_c e^{S\tau} \quad \Phi_j^*(-S) \quad H_i(S) \quad \Phi_k(S) \frac{dS}{2\pi j}, \tag{3-26}$$

where τ is a delay variable.

The reader may recognize this equation as the transform of the convolution integral, i.e.,

$$h_{ji}(\tau) = \int_{-\infty}^{+\infty} h_j(t) \quad f_i(\tau - t) \quad dt \tag{3-27}$$

where $h_j(t)$ is the weighting function of the filter $H_i(S)\ \Phi_j^*(-s)$ and $f_i(t)$ is the $i^{\underline{th}}$ input signal.

Letting $\tau = 0$; the equation gives the value of a sample of the output signal at $\tau = 0$. Thus we obtain,

$$h_{jk} = \int_{-\infty}^{+\infty} h_j(t)\, f_i(t)\, dt = \int_C \Phi_j^*(-S)\, H_i(S)\, \Phi_k(S)\, \frac{dS}{2\pi j}. \tag{3-28}$$

These values of h_{jk} (now) represent the results of the input signal acting on the transfer function $H_i(S)$ and the measuring filter $\Phi_j^*(-S)$ at the time $\tau = 0$. Forming the H_α matrix with these values of h_{jk} as the elements, we have:

$$H_\alpha = \begin{array}{cc} & k \longrightarrow \\ \begin{array}{c} j \\ \downarrow \end{array} & \begin{bmatrix} h_{11} & h_{12} & --- & h_{1n} \\ h_{21} & h_{22} & --- & h_{2n} \\ \vdots & & & \vdots \\ h_{m1} & --- & & h_{mn} \end{bmatrix} \end{array} \tag{3-29}$$

where each row represents the output of a filter and each column reflects an input signal.

Now form a column matrix C

$$\begin{bmatrix} C_1 \\ \vdots \\ C_n \end{bmatrix} \tag{3-30}$$

which represents the magnitude of the probing signal components. An additional requirement from practical considerations is that energy be constant, viz,

$$\sum_{n=1}^{N} C_n^2 = 1. \tag{3-31}$$

Multiplying the H_α matrix by this column matrix [C] will give a column matrix

$$M_\alpha = [H_\alpha]\,[C] \tag{3-32}$$

which is the representative of the signal appearing at the output of each filter due to the partial system $H_i(S)$.

The collection of these columns may be arranged to form a matrix, M, called the modulation matrix.

$$M = [M_o, M_1, \,.\, M_\alpha \,.\, M_n] \tag{3-33}$$

By arranging the parameter deviations as a column array

$$[A] = \begin{bmatrix} \alpha_o \\ \alpha_1 \\ \cdot \\ \cdot \\ \cdot \\ \alpha_n \end{bmatrix} \tag{3-34}$$

The total system response, G, can be represented as

$$[G] = [M] \,.\, [A] = [M_o]\,\alpha_o + [M_1]\,\alpha_1 + \,.\,.\,.\,. \tag{3-35}$$

The values of α_o through α_n can be determined by solving this matrix equation, i.e.,

$$[M^{-1}]\,[G] = [A] \tag{3-36}$$

Since $[M^{-1}]$ is composed of the input signal magnitude [C], these magnitudes can be adjusted to maximize the estimate of the parameter when they are subjected to noise.

The minimization of white noise can be accomplished from least-square statistical theory. The criterion selected for optimization is the minimization of the covariance of the error in the parameter estimates, as expressed by the minimum variance estimator

$$(\bar{M}\ M)^{-1}$$

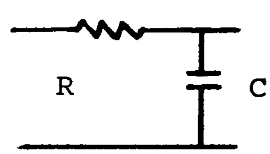
This can be minimized by maximizing the determinate

$$\left|\bar{M}\ M\right| = \left|M\right|^{2} \tag{3-37}$$

3.6 COMPONENT VALUES AND TRANSFER FUNCTION PARAMETERS

Testing the transfer function coefficients allows determination of system parameters, which are important to proper action. Also, the determination of component values, can be obtained from the transfer function coefficients. Each of the coefficients are related by a linear equation to the component values in a circuit. For example, take the circuit illustrated in Figure 3-4 and analyze the transfer function in terms of the component values:

Figure 3-4



R-C Circuit

The transfer function is

$$H(S) = \frac{c_o}{d_1 S + d_o} = \frac{1/CS}{\frac{1}{CS} + R} = \frac{1/C}{RS + 1/C} \tag{3-38}$$

The measurement of the coefficient d_1 will determine R, while the measurement of the coefficient d_o, will determine 1/C.

$$d_o = \frac{1}{C}$$
$$d_1 = R \tag{3-39}$$

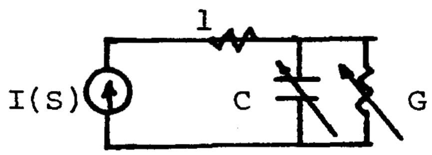
For more complex transfer functions, the same results hold. Generally,

$$d_n = f_n \ (R, C, L) \text{ and,}$$
$$c_n = f_n \ (R, C, L). \tag{3-40}$$

The solution of these equations will determine the component values of the circuit.

3.7 ILLUSTRATIVE EXAMPLE

The example used to demonstrate the method of growing exponentials, is illustrated in the following circuit diagram.



R-C Circuit

Figure 3-5

This circuit has an impedance, $Z(S)$, of

$$\frac{E(S)}{I(S)} = Z(S) = 1 + \frac{1}{CS + G} = 1 + \frac{1}{d_1 S + d_o} \tag{3-41}$$

To test this complex impedance transfer function, first find the partial derivatives of this transfer function, with respect to the parameters to be tested. The parameters to be chosen are the coefficients of the denominator, i.e., C, and G. Let the nominal values of C and G be unity, the partial systems become

$$\frac{\partial Z(S)}{\partial C} = \frac{-S}{(S + 1)^2} \tag{3-42}$$

$$\frac{\partial Z(S)}{\partial G} = \frac{-1}{(S + 1)^2} \qquad (3\text{-}43)$$

To explore or test these partial systems, make use of two orthogonal probing signals which match the partial system. Notice that the partial systems are orthogonal and therefore, independent. The probing signal is determined by using the method described by Kautz. This example uses the Kautz relation, Equation 3-3.

The representatives of the backward orthogonal probing signals in the complex plane are

$$\Phi_1(S) = \sqrt{2} \quad \frac{1}{-S + 1} \qquad (3\text{-}44)$$

$$\Phi_2(S) = \sqrt{2} \quad \frac{-S -1}{(-S - 1)^2} \qquad (3\text{-}45)$$

In the time domain these signals are

$$f_1(t) = \sqrt{2} \quad e^{t} \quad \text{for } t < 0 \qquad (3\text{-}46)$$

$$f_2(t) = \sqrt{2} \quad e^{t} \quad (1 + 2t),\ t < 0 \qquad (3\text{-}47)$$

The matrix representative on a backward basis of the two component systems for the two parameter system, is found by applying equation 3-28. Which gives the relationships

$$H_G = \begin{bmatrix} -\frac{1}{4} & \frac{1}{2} \\ 0 & -\frac{1}{4} \end{bmatrix} \qquad H_C = \begin{bmatrix} -\frac{1}{4} & 0 \\ 0 & -\frac{1}{4} \end{bmatrix} \qquad (3\text{-}48)$$

The optimum probing signal components are found by forming the columns given by equation 3-32

$$M_G = H_G \begin{bmatrix} \cos \Psi \\ \sin \Psi \end{bmatrix} = \tfrac{1}{4} \begin{bmatrix} 2 \sin \Psi - \cos \Psi \\ - \sin \Psi \end{bmatrix} \tag{3-49}$$

$$M_C = H_C \quad [c] = \tfrac{1}{4} \begin{bmatrix} - \cos \Psi \\ - \sin \Psi \end{bmatrix} \tag{3-50}$$

The modulation matrix is formed by the collection of the M_α columns, thus,

$$M = \tfrac{1}{4} \begin{array}{cc} \begin{bmatrix} - \cos \Psi & 2 \sin \Psi - \cos \Psi \\ - \sin \Psi & - \sin \Psi \end{bmatrix} & \begin{array}{c} 1 \\ 2 \end{array} \\ \begin{array}{cc} C \qquad\qquad & \qquad\qquad G \end{array} & \end{array} \tag{3-51}$$

The maximum value of the determinate

$$|\overline{M}\, M| = |M|^2 \alpha \sin \Psi \tag{3-52}$$

occurs when

$$\Psi = \pi/2 \tag{3-53}$$

The optimum probing signal is

$$\sqrt{2}\ (1 + 2t)\ e^{t} \qquad \text{for } t < 0 \tag{3-54}$$

The estimator matrix M^{-1} is

$$M^{-1} = \begin{array}{cc} \begin{array}{cc} 1 & \quad 2 \end{array} & \\ \begin{bmatrix} -2 & -4 \\ 2 & 0 \end{bmatrix} & \begin{array}{c} C \\ G \end{array} \end{array} \tag{3-55}$$

3.8 GENERALIZED SOLUTION OF FIRST ORDER TRANSFER FUNCTIONS

The general first order transfer function is

$$H(S) = \frac{c_1(S) + c_o}{d_1(S) + d_o} \tag{3-56}$$

By differentation of this function with respect to the coefficients, we obtain

$$\frac{\partial H(S)}{\partial c_1} = H_{c_1}(S) = \frac{S}{d_1 S + d_o} = \frac{1}{d_1} - \frac{d_o/d_1}{d_1 S + d_o} \tag{3-57}$$

$$\frac{\partial H(S)}{\partial c_o} = H_{c_o}(S) = \frac{1}{d_i S + d_o} \tag{3-58}$$

$$\frac{\partial H(S)}{\partial d_1} = H_{d_1}(S) = \frac{-S\ (c_1 S + c_o)}{\{d_1 S + d_o\}^2} \tag{3-59}$$

$$\frac{\partial G(S)}{\partial d_o} = H_{d_o}(S) = \frac{c_1 S + c_o}{\{d_1 S + d_o\}^2} \tag{3-60}$$

Two testing exponentials were determined by the Kautz relation. These became

$$\Phi_1(s) = \sqrt{2\ d_{o/d_1}} \quad \frac{1}{-S + \frac{d_o}{d_1}} \tag{3-61}$$

$$\Phi_2(S) = \sqrt{2\ d_{o/d_1}} \quad \frac{-S - d_o/d_1}{(-S + d_o/d_1)^2} \tag{3-62}$$

where $\Phi(S)$ is the representation of this negative time function in the frequency domain.

In the time domain

$$f_1(t) = \sqrt{2\, d_o/d_1}\; e^{\frac{d_o}{d_1} t} \qquad \text{for } t < 0 \tag{3-63}$$

$$f_2(t) = \sqrt{2\, d_o/d_1}\; \{1 + 2t\}\, e^{\frac{d_o}{d_1} t} \qquad \text{for } t < 0 \tag{3-64}$$

An approximation to these growing exponentials was found to operate successfully on the analog computer.

$$f_1(t) \approx \sqrt{2\, d_o/d_1}\; e^{\frac{d_o}{d_1}\left[t - 5\frac{d_1}{d_o}\right]} \qquad \text{for } 0 \le t \le 5\,\frac{d_1}{d_o} \tag{3-65}$$

$$f_2(t) \approx \sqrt{2\, d_o/d_1}\; \left[1 + 2\left\{t - 5\frac{d_1}{d_o}\right\}\right] e^{\frac{d_o}{d_1}\left[t - 5\frac{d_1}{d_o}\right]} \tag{3-66}$$

for $0 \le t \le 5\,\dfrac{d_1}{d_o}$

The matrix representation of the signals appearing at the filter outputs of the partial systems are:

$$H_{c_1} = \begin{bmatrix} +\frac{1}{2d_1}, & \frac{1}{d_1} \\ 0 & -\frac{1}{2d_1} \end{bmatrix} \tag{3-67}$$

$$H_{c_o} = \begin{bmatrix} +\frac{1}{2d_o}, & -\frac{1}{2d_1} \\ 0 & +\frac{1}{2d_o} \end{bmatrix} \tag{3-68}$$

$$H_{d_1} = - \begin{bmatrix} \dfrac{c_1 d_o + c_o d_1}{4 d_1^{2} d_o}, & -\frac{1}{2} \dfrac{c_1}{d_1^{2}} \\ & \\ 0 & \dfrac{c_1 d_o + c_1}{4 d_1^{2} d_o} \end{bmatrix} \tag{3-69}$$

$$H_{d_o} = - \begin{bmatrix} \dfrac{d_o c_1 + c_o d_1}{4 d_o^{2} d_1}, & +\frac{1}{2} \dfrac{c_o}{d_o^{2}} \\ & \\ 0 & + \dfrac{c_1 d_o + c_o d_1}{4 d_o^{2} d_1} \end{bmatrix} \tag{3-70}$$

Forming the matrix M and maximizing its determinant leads to the result that only the second signal $f_2(t)$ is required.

$$\begin{aligned} C_1 &= 0 = \cos \Psi \\ & \qquad\qquad \Psi = \pi/2 \\ C_2 &= 1 = \sin \Psi \end{aligned} \tag{3-71}$$

The estimators for the parameters c_n and d_n are (3-72)

$$M_d^{-1} = \frac{-2\, d_o^{3}\, d_1^{3}}{c_1^{2} d_o^{2} - c_o^{2} d_1^{2}} \begin{array}{cc} \begin{array}{cc} 1 & 2 \end{array} & \\ \begin{bmatrix} \dfrac{c_1 d_o + c_o d_1}{4 d_o^{2} d_1}, & + \dfrac{c_o}{d_o^{2}} \\ & \\ \dfrac{c_1 d_o + c_o d_1}{d_1^{2} d_o}, & -2 \dfrac{c_1}{d_1^{2}} \end{bmatrix} & \begin{array}{c} \Delta d_1 \\ \\ \\ \Delta d_o \end{array} \end{array}$$

$$M_c^{-1} = \begin{array}{cc} \begin{array}{cc} 1 & 2 \end{array} & \\ \begin{bmatrix} \dfrac{4d_1}{6}, & \dfrac{4d_1}{6} \\ & \\ \dfrac{-4d_o}{6}, & \dfrac{+4d_o}{3} \end{bmatrix} & \begin{array}{c} \Delta c_1 \\ \\ \\ \Delta c_o \end{array} \end{array} \tag{3-73}$$

Normalizing the estimator is accomplished by dividing each row by the parameter which that row estimates.

$$M_d^{-1} = \begin{array}{c} \begin{array}{cc} \qquad\qquad 1 \qquad\qquad & \qquad\qquad 2 \qquad\qquad \end{array} \\ \begin{bmatrix} -\dfrac{2d_o\, d_1}{c_1\, d_1 - c_o\, d_1} & -\dfrac{4c_o\, d_o\, d_1^{\,2}}{c_1^{\,2}\, d_o^{\,2} - c_o^{\,2}\, d_1^{\,2}} \\[3ex] -\dfrac{2d_o\, d_1}{c_1\, d_1 - c_o\, d_1} & +\dfrac{4c_1\, d_1\, d_o^{\,2}}{c_1^{\,2}\, d_o^{\,2} - c_o^{\,2}\, d_1} \end{bmatrix} \end{array} \begin{array}{c} \Delta d_1/d_1 \\[6ex] \Delta d_o/d_o \end{array} \tag{3-74}$$

$$M_c^{-1} = \begin{array}{c} \begin{array}{cc} \quad 1 \quad & \quad 2 \quad \end{array} \\ \begin{bmatrix} \frac{2}{3}\,\dfrac{d_1}{c_1} & \frac{2}{3}\,\dfrac{d_1}{c_1} \\[3ex] -\frac{2}{3}\,\dfrac{d_o}{c_o} & +\frac{4}{3}\,\dfrac{d_o}{c_o} \end{bmatrix} \end{array} \begin{array}{c} \Delta c_1/c_1 \\[6ex] \Delta c_o/c_o \end{array} \tag{3-75}$$

Thus, using this result for estimation and $f_2(t)$ as a probing signal, any first order transfer function can be measured to obtain $\frac{\Delta d_1}{d_1}$, $\frac{\Delta d_o}{d_o}$, $\Delta c_1/c_1$, and $\Delta c_o/c_o$ with one probing signal.

3.9 EXPERIMENTATION

The growing exponential method was the first technique to be investigated experimentally. The problems chosen for experimentation were

$$H(S) = \frac{1}{CS + G} \quad \text{and} \quad H(S) = \frac{1.5S + 1S}{S + 1}\,. \tag{3-76}$$

The probing signals for these circuits were generated and supplied to an analog simulation. Various methods of generating these signals were tried, and one proved to be successful. This method consisted of time shifting the time axis, so that we could generate the rising exponentials with controlled unstable circuits.

For example, suppose we wished to simulate the signal.

$$f_1(t) = \sqrt{2\, {}^{d_o}/{}_{d_1}}\; e^{\frac{d_o}{d_1}(t)} \quad \text{for } t < 0. \tag{3-77}$$

First we shifted the time axis by <u>five</u> time constants to obtain,

$$f(t - 5\, {}^{d_1}/{}_{d_o}) = \sqrt{2\, {}^{d_o}/{}_{d_1}}\; e^{\frac{d_o}{d_1}(t - 5\, {}^{d_o}/{}_{d_1})} \quad \text{for } 0 < t < 5\,\frac{d_1}{d_o} \tag{3-78}$$

Notice that the result is a <u>positive</u> time function. This time function was then generated on the analog computer. We were greatly concerned that this approximation would give bad results. However, it turned out that the results were very good, and the approximation apparently had very little effect.

In continuing our effort to experimentally test the transfer function, we constructed two models of the system. One model represented the normal transfer function. The second model represented the system in which the parameters could be varied. Then, we simulated the filters, control, and sampling logic. Figure 3-6 illustrates the simulation set up on the analog computer. Notice from the figure that the simulation includes sufficient generality to be applicable to two examples.

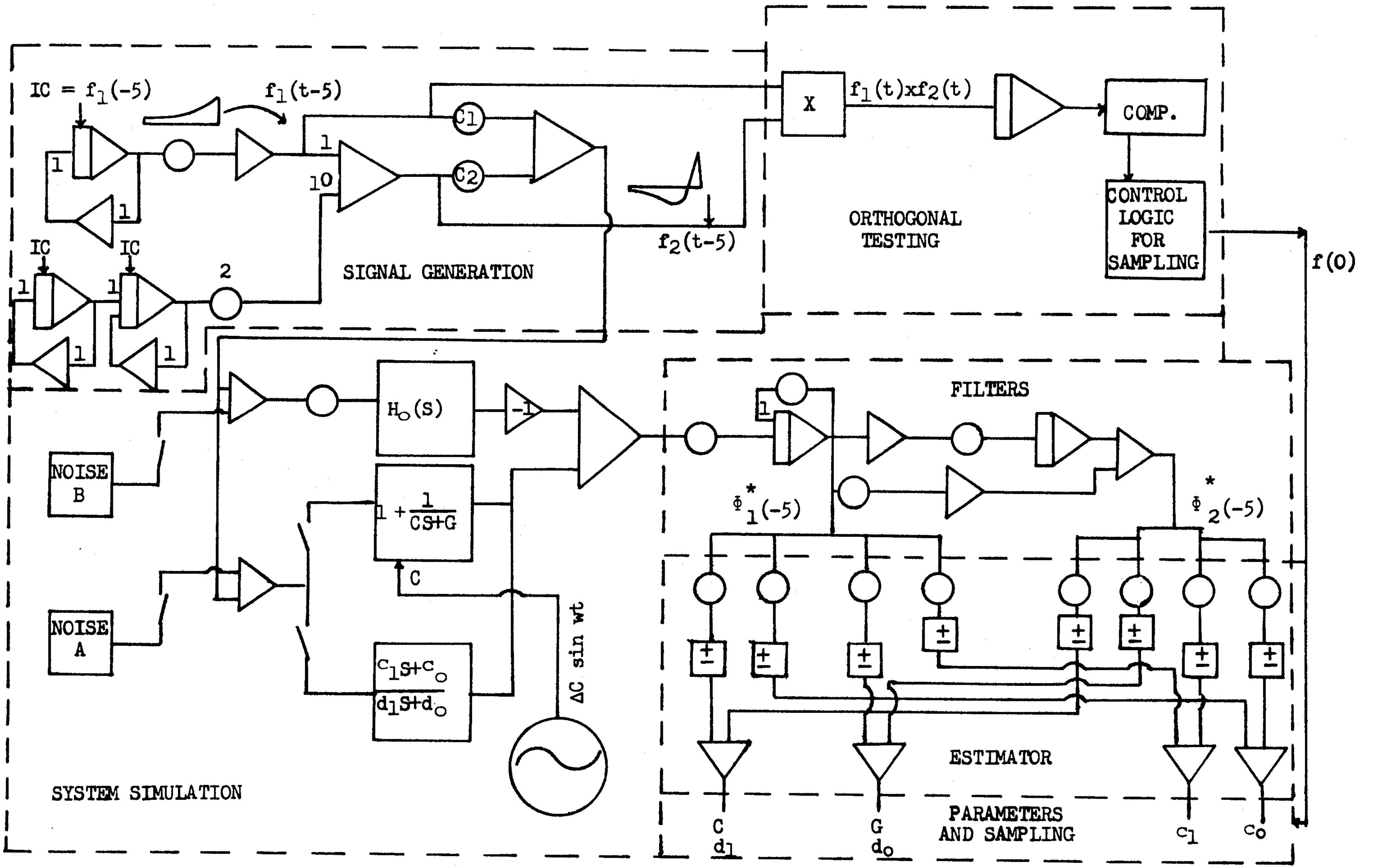


Figure 3-6

Computer Simulation First and Second Example

3.9.1 The First Example

The first transfer function,

$$H(S) = \frac{1}{SC + G} \tag{3-79}$$

was tested. Results of the estimator as a function of time is given in Figure 3-7. For parameter increments of 3%, i.e., 3%, 6%, 9%, 12%, 15%, 18%, etc. The measurement of the parameters were taken when the probing signal stopped. Notice that when only one parameter is varied, the estimate of the other signal is approximately zero. Notice also that an incremental change in a parameter results in a similar incremental change in the estimate. The estimates were good up to approximately a 40% change in the parameter. After 40%, there was a considerable interaction between the estimators for C and G. This was not a surprising result, because our approximation of the transfer function was based on a Taylor Series Expansion, which is only accurate within a limited region, when all the higher order terms are neglected.

3.9.1.1 Noise Experiments

After concluding this test and establishing that we could measure the changes in C and G, we decided a noise analysis was necessary. The noise analysis was conducted by inserting independent band limit noise into both the nominal and actual system under test. We then applied the input signal and observed the results of measuring a parameter variation of 10% in the capacitor, C. Signal to noise ratio was measured by the ratio

$$\frac{\text{peak voltage of signal}}{\text{rms noise voltage}}$$

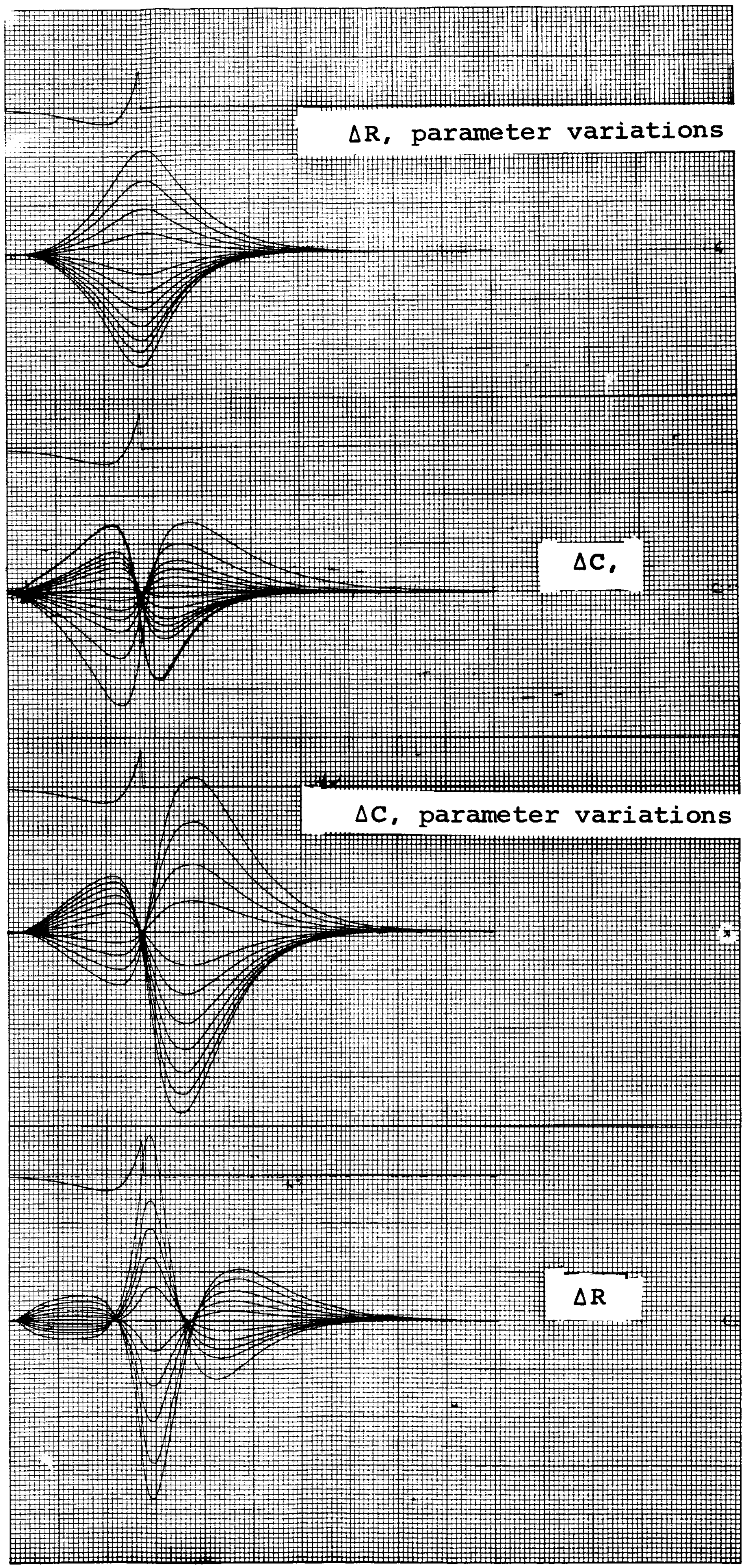


Figure 3-7 ΔR Parameter variations with no variations in ΔC and ΔC Parameter with no variations in ΔR.

From this noise analysis, (Figures 3-8,9,10) it was concluded that the testing signal should be greater than 26 db in voltage above the rms noise level of either the nominal or measured system. With a 26 db voltage ratio the range of indication of C was 10% ± 1.5%.

3.9.1.2 Time Varying Parameters

To study the effects of time varying parameters, the capacitance C was allowed to vary sinusoidally, as

$$C + \Delta C \sin \omega_o t.$$

The radian frequency ω_o was varied and the indications of ΔC observed. A 10% change in the parameter, C, was allowed, and the frequency was induced at ω_o = .25, .5, 1., 2., 3.77, 10 and 100, (Figure 3-11). At ω_o = 3.77 the amplitude of C was varied over the values, 5%, 10%, and 20%, (Figure 3-12). The results of this study show that in tests of time varying parameters, good indications can be measured at radian frequency below one half of the location of the transfer function pole.

3.9.2 The Second Example

The second transfer function was tested by using the general relationship developed for

$$\frac{c_1 S + c_o}{d_1 S + d_o} = \frac{1.5S + 15}{S + 1}$$

where

$$c_1 = 1.5 \qquad d_1 = 1$$

$$c_o = 15 \qquad d_o = 1$$

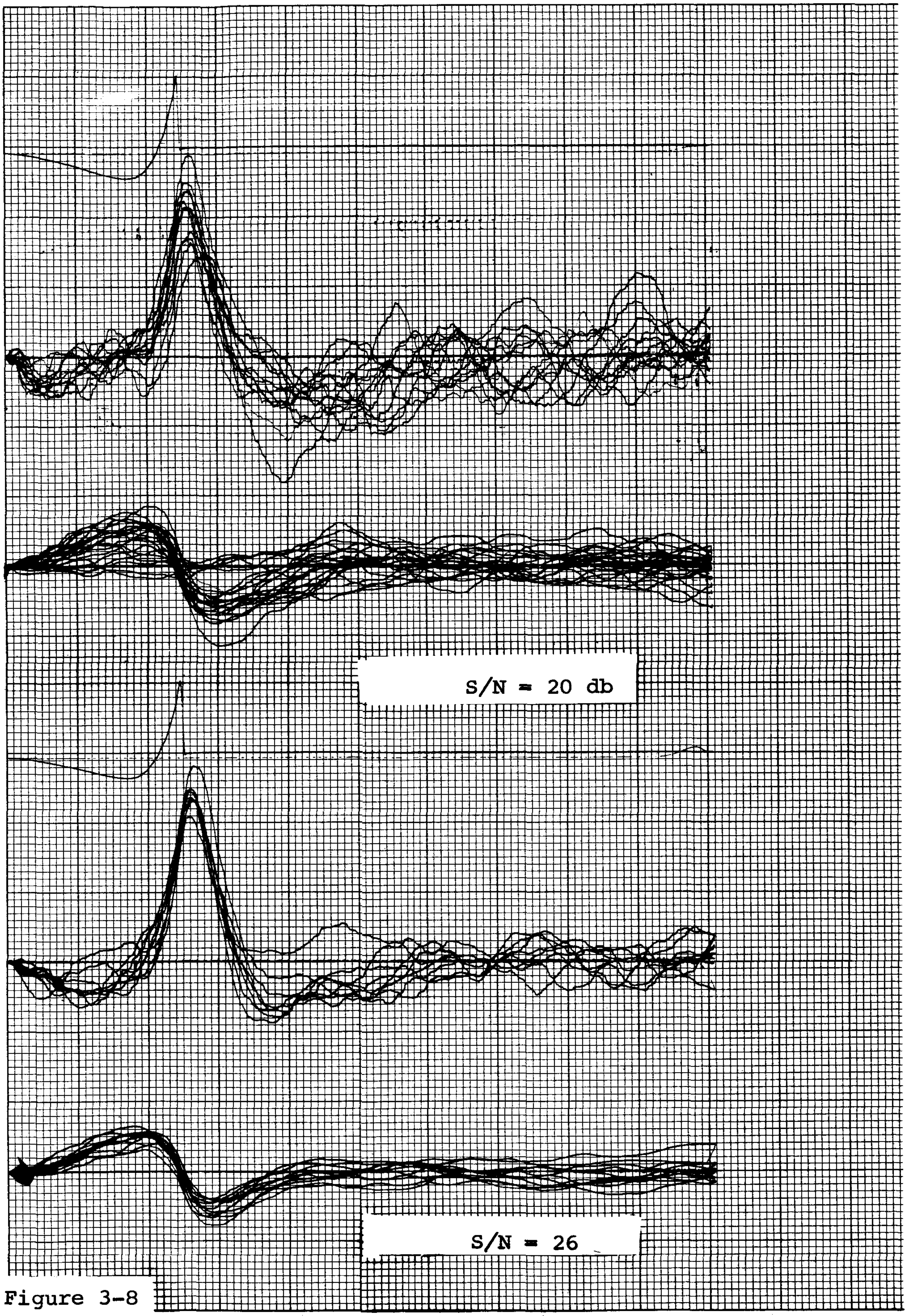


Figure 3-8

Noise Analysis, ΔC = .1, Varying Noise Level

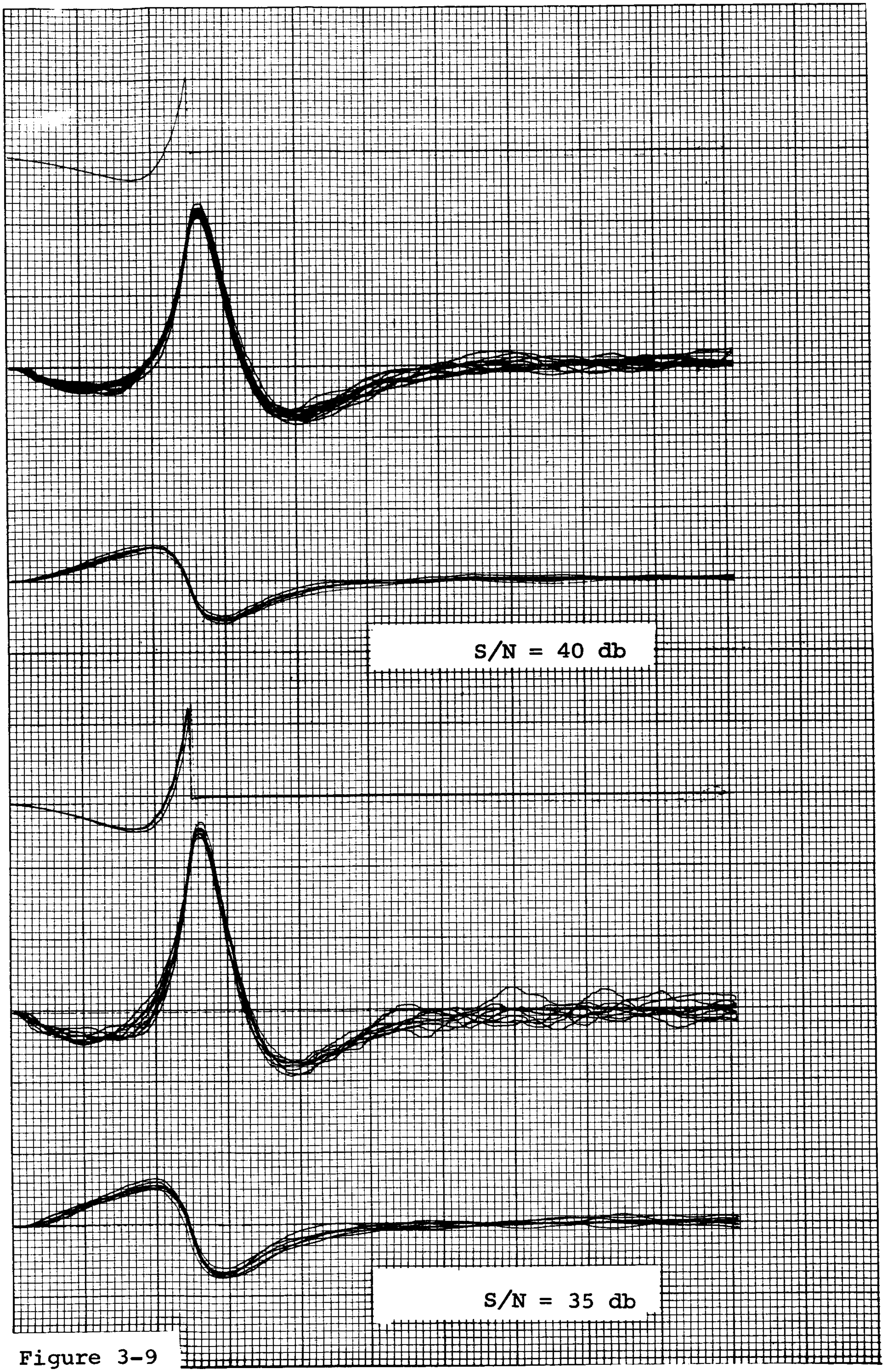


Figure 3-9

Noise Analysis, ΔC = .1, Varying Noise Level

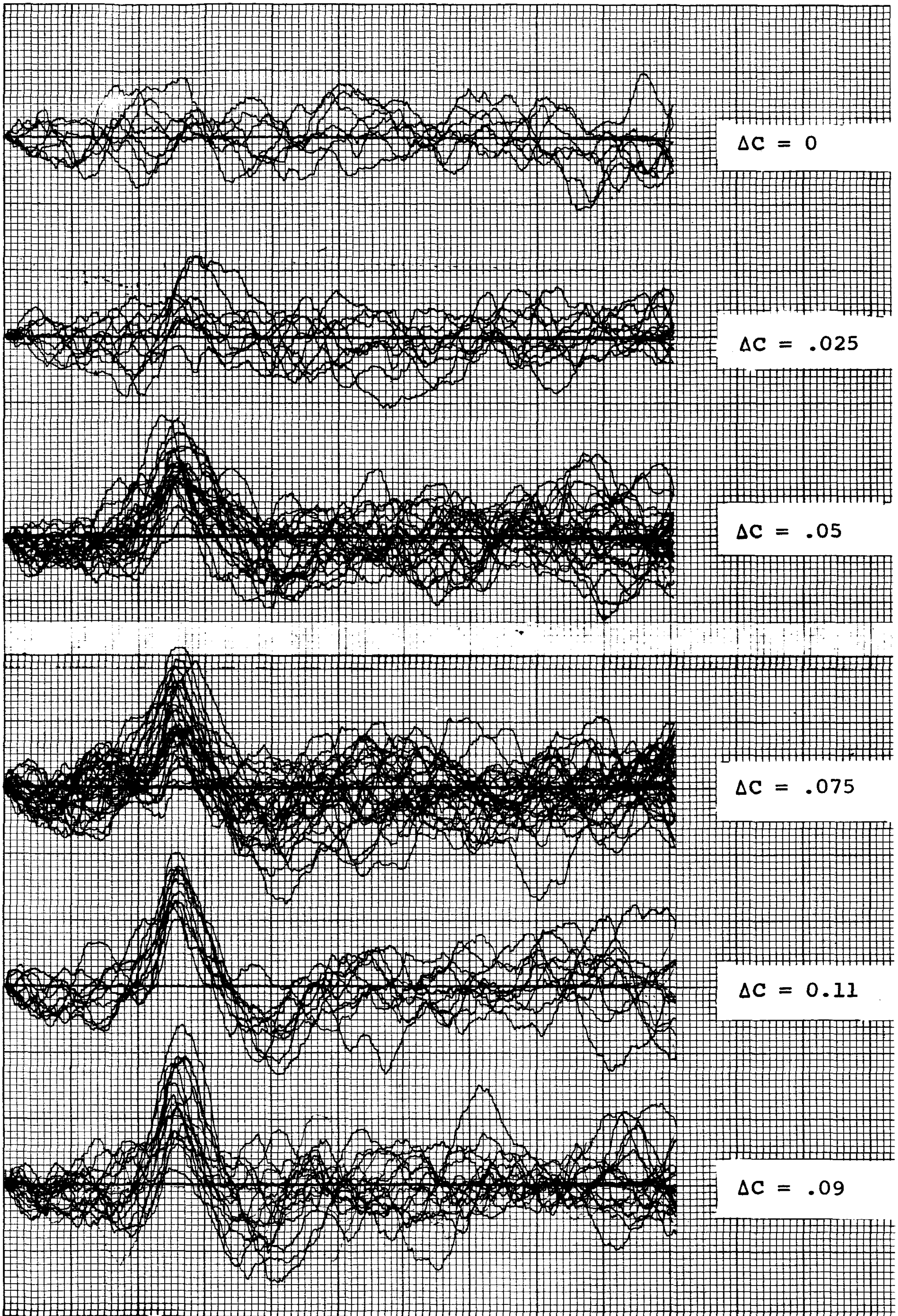


Figure 3-10 Noise Analysis varying ΔC, with S/N = 20 db

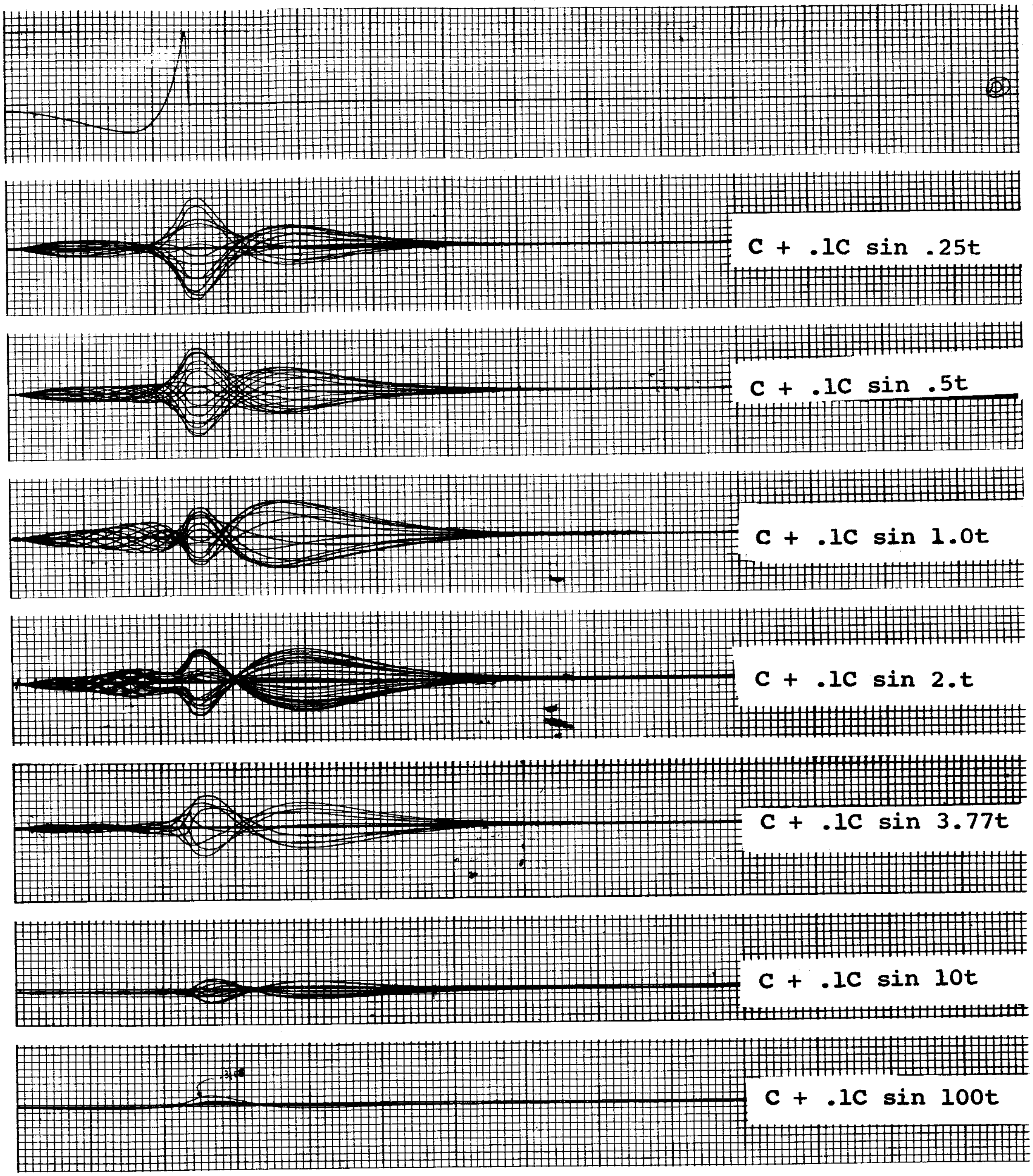


Figure 3-11 Time varying parameter analysis

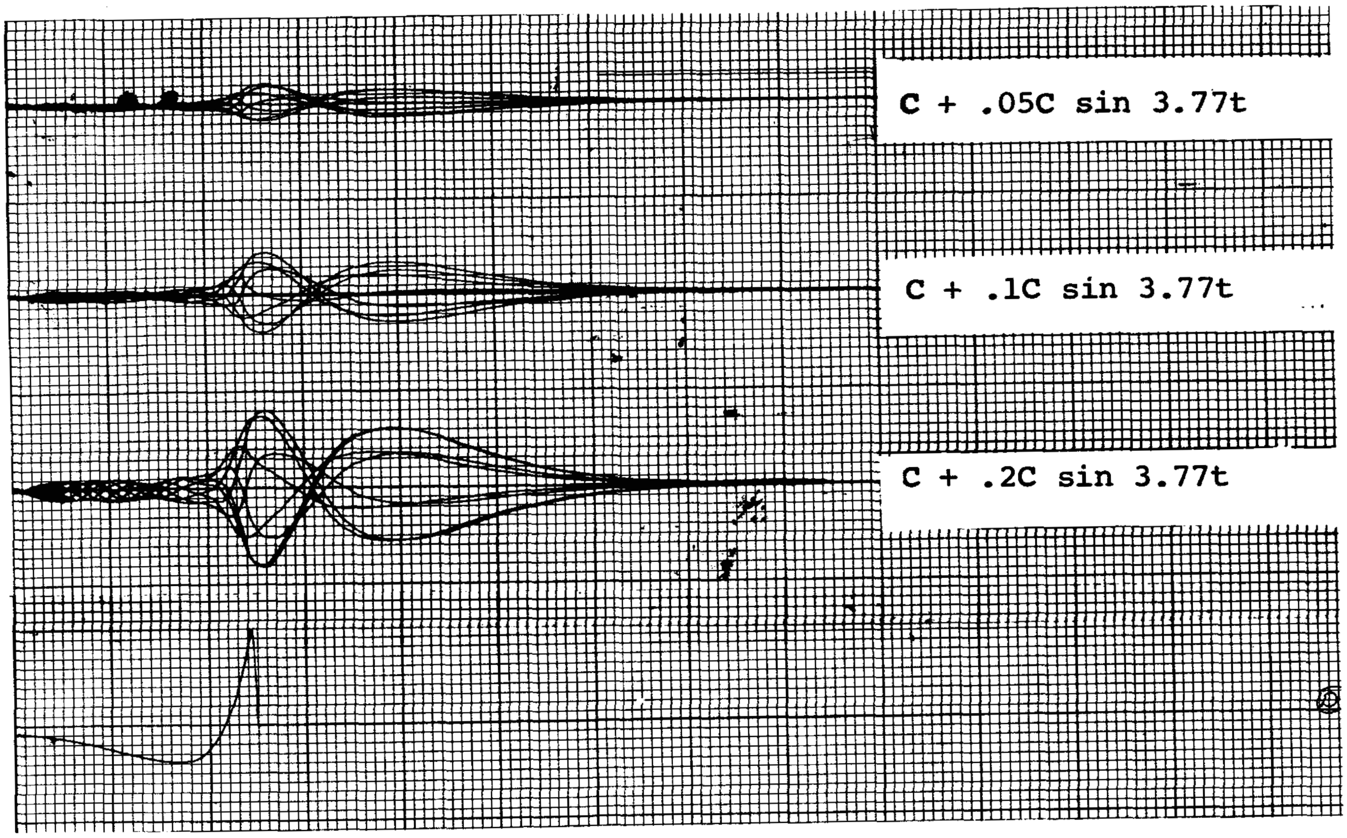


Figure 3-12   Time varying parameter analysis

The experiment was connected on this transfer function to establish the measurability of coefficients in the numerator and denominator. The previous example only measured the effect of coefficients in the denominator.

The probing signal was applied and the estimates of $\Delta c_o/c_o$, $\Delta c_1/c_1$ and $\Delta d_o/d_o$, $\Delta d_1/d_1$ were made. The results are given in Figures 3-13, 3-14. In these figures the outputs of the estimators are plotted against each other. The time function was stopped when the input probing signal was shut off. The lobing on the time traces give the value of a normalized change in each parameter. Each line on the graph is a 1% change. The results of the estimation were orthogonal as expected. A percentage change in one parameter resulted in an indication of that parameter and practically no change in the other.

There was a considerable amount of interaction between the indications of the c's and d's. Notice in Figure 3-15 that a - 10% variation in c_o and d_o resulted in a measurement of

$$\frac{\Delta d_o}{d_o} = + .037$$

$$\frac{\Delta d_1}{d_1} = + .1$$

$$\frac{\Delta c_o}{c_o} = - .05$$

$$\frac{\Delta c_1}{c_1} = .15$$

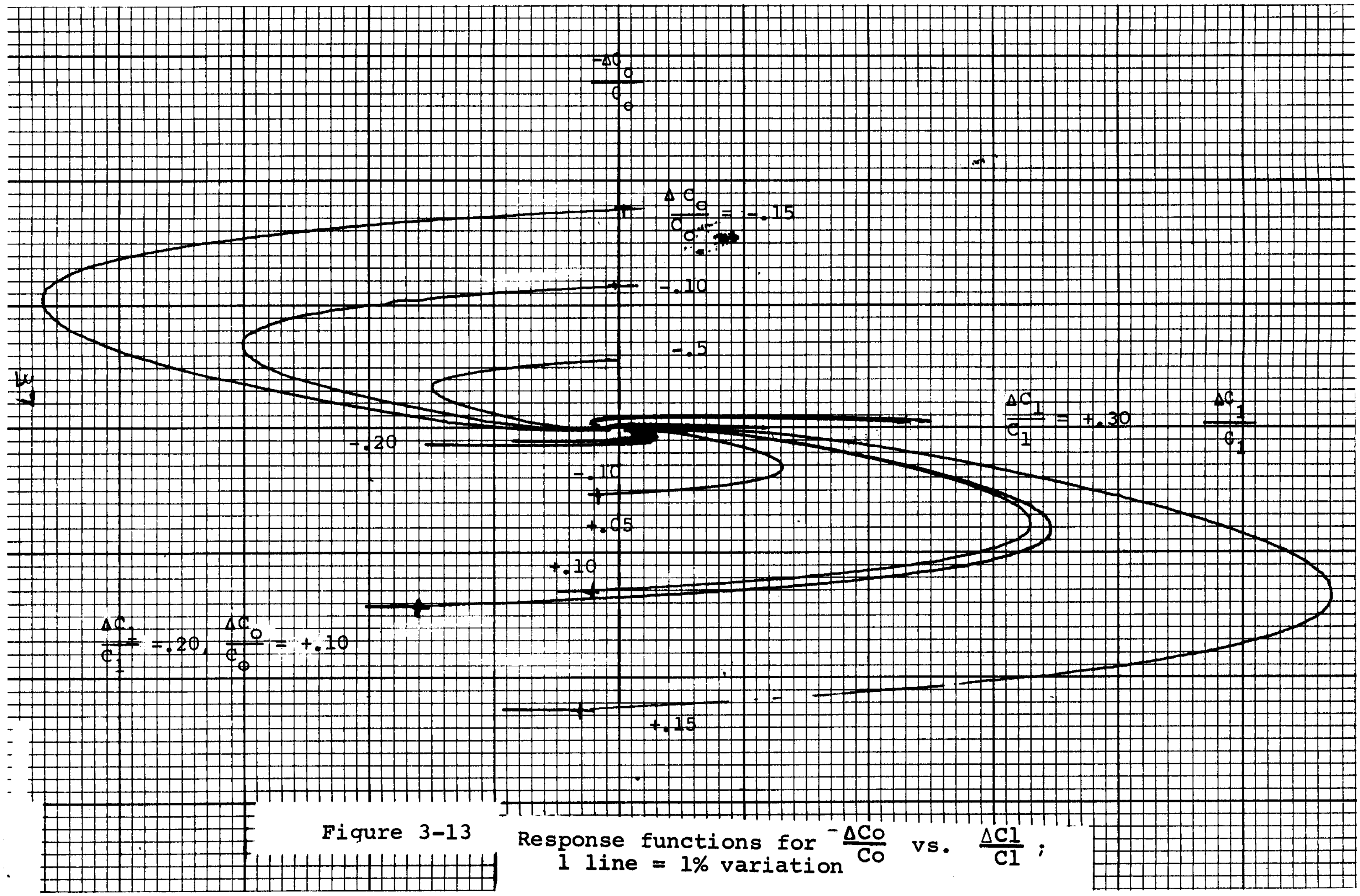


Figure 3-13 Response functions for $\frac{-\Delta Co}{Co}$ vs. $\frac{\Delta C1}{C1}$;
1 line = 1% variation

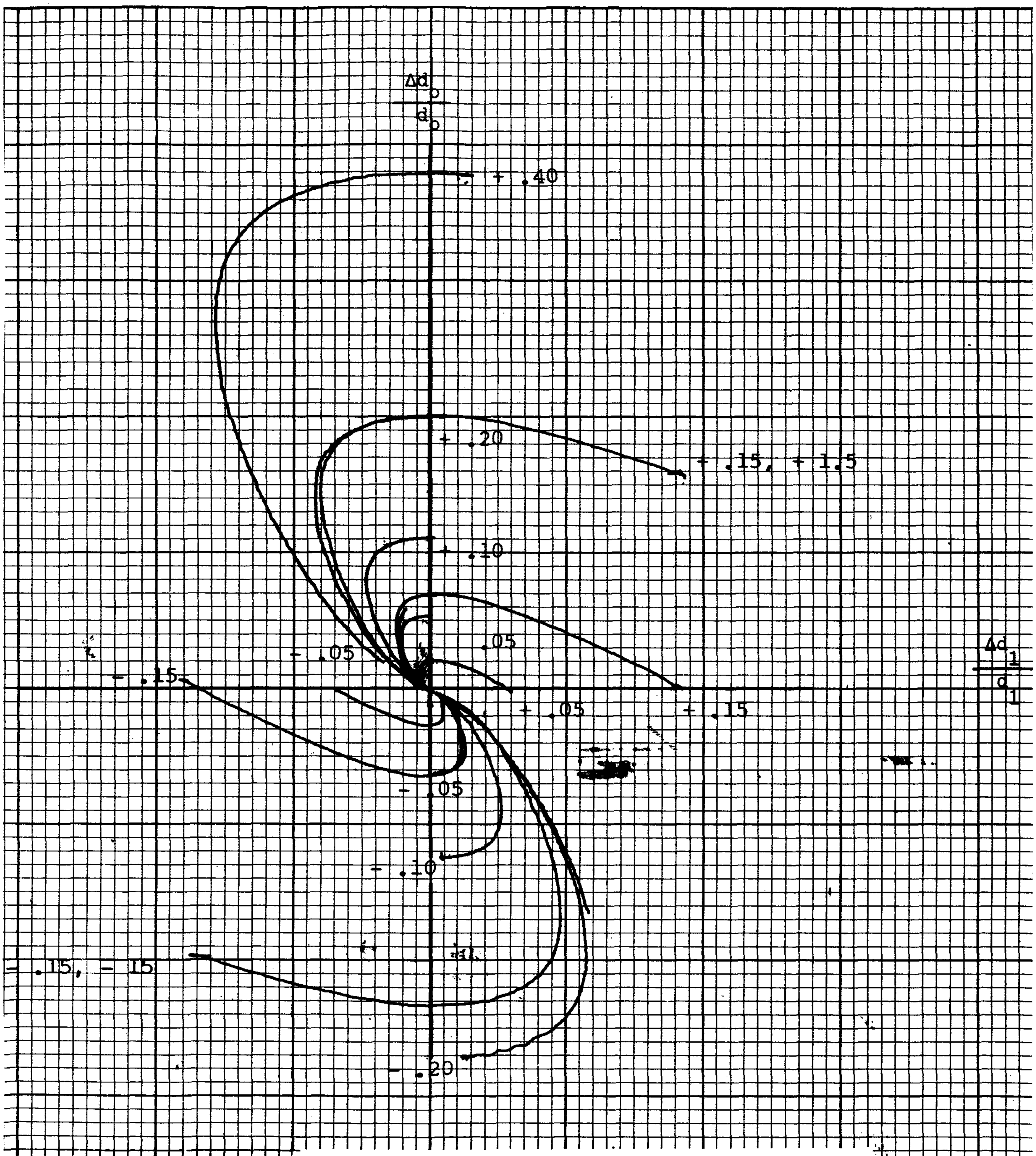


Figure 3-14 Response functions for $\frac{\Delta do}{do}$ vs. $\frac{\Delta dl}{dl}$; 1 line = 1% variation

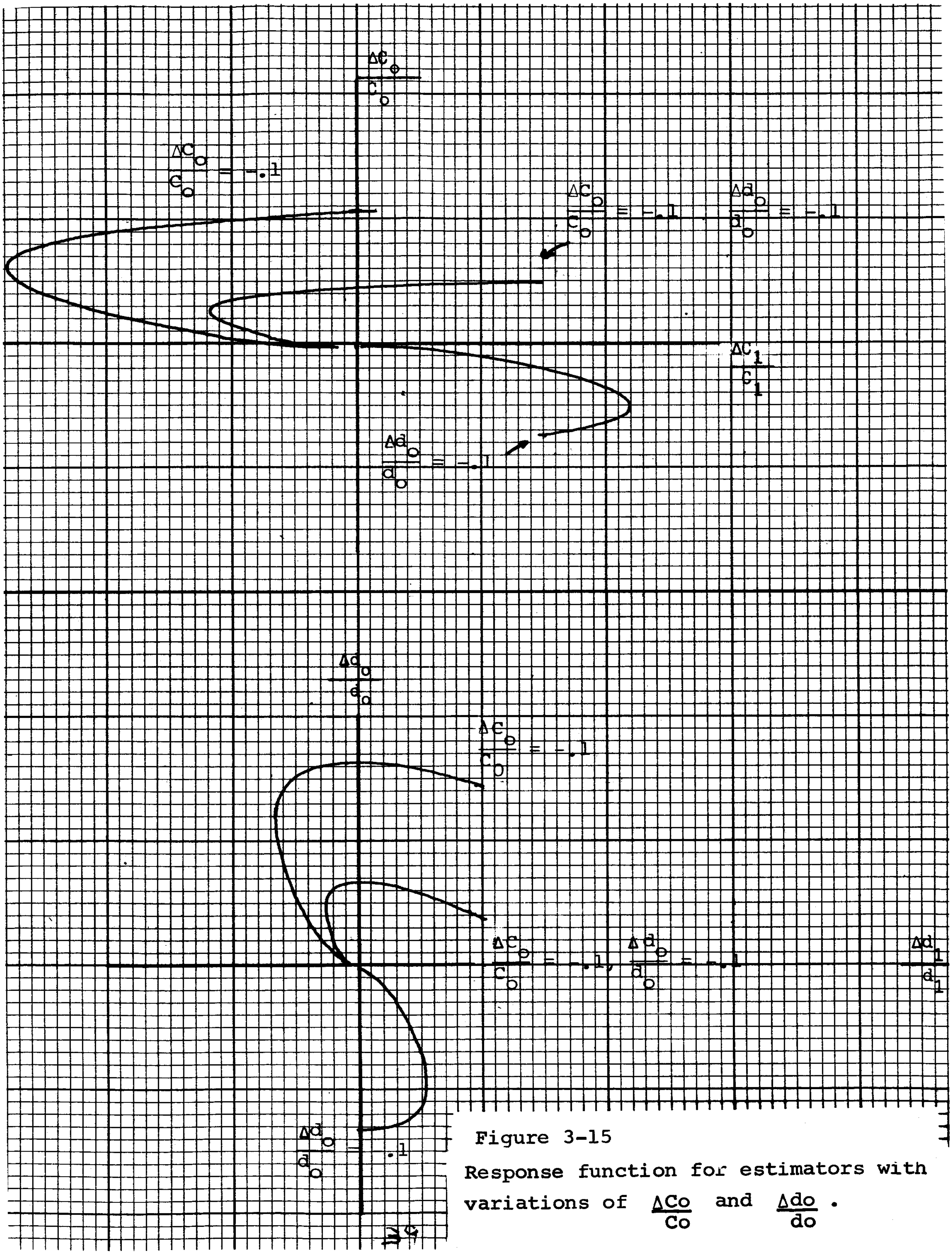


Figure 3-15

Response function for estimators with variations of $\frac{\Delta Co}{Co}$ and $\frac{\Delta do}{do}$.

The results of the measurements on c and d indicated that the transfer function was

$$\frac{1.725S + 14.25}{1.1S + 1.037} = \frac{1.57S + 13.0}{S + .94}$$

Thus, there was approximately a

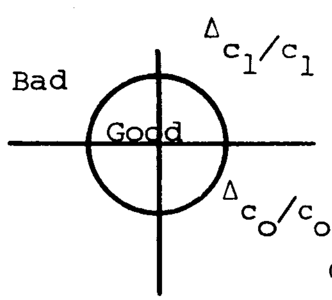
+2% error in measuring c_1

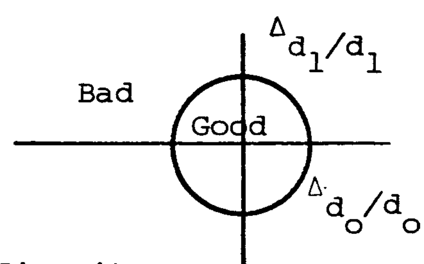
+4% error in measuring d_o and

-4% error in measuring c_o.

The results are favorable and seem to allow decision on a go-no go basis. If the tolerance limitation can be established on all of the parameters, then experimental data can be taken to establish limits on the values of the estimator.

For example, if 10% tolerances are imposed on all the parameters, then a circle in both plots, as illustrated below would separate a good circuit from a bad circuit.





Good/Bad Circuits
Figure 3-16

If an indication was bad from <u>both</u> estimators, the circuit could be considered bad.

3.10 SECOND ORDER TRANSFER FUNCTIONS

The analysis of a second order transfer function was conducted. In this analysis the testing of only two parameters were investigated to determine applicability of the growing exponential method. All of the math is included to illustrate the complexity and length of the method. The second order transfer function chosen was

$$H(S) = \frac{c_1 S + c_o}{S^2 + d_1 S + d_o} = \frac{S + 5}{S^2 + 2S + 10}$$

This transfer function can also be written as

$$H(S) = \frac{c_1 S + c_o}{(S + \alpha)^2 + \beta^2}$$

The partial derivatives of H(S) with respect to the parameters c_1 and c_o are the component partial systems corresponding to c_o and c_1:

$$\frac{\partial H}{\partial c_1} = \frac{S}{(S + \alpha)^2 + \beta^2} = H_{c_1}(S)$$

$$\frac{\partial H}{\partial c_o} = \frac{1}{(S + \alpha)^2 + \beta^2} = H_{c_2}(S)$$

From complex poles at $S_v = -\alpha_v - j\beta_v$ and $\overline{S}_v = -\alpha_v + j\beta_v (v=1,2,\ldots, \frac{n}{2})$ the backward time probing signals are found from the Kautz relation,

$$\frac{\Phi_{2v-1}(S)}{\Phi_{2v}(S)} = \sqrt{2\alpha_v}\;\frac{\left[(S - \alpha_1)^2 + \beta_1^{\,2}\right]\ldots\left[(S - \alpha_{v-1})^2 + \beta_{v-1}^{\,2}\right](S \pm |S_v|)}{\left[(S + \alpha_1)^2 + \beta_1^{\,2}\right]\ldots\left[(S + \alpha_{v-1})^2 + \beta^2_{v-1}\right]\left[(S+\alpha_v)^2 + \beta_v^{\,2}\right]}$$

where v = 1, 2 - - - n/2.

The upper (plus) sign pertains to Φ_{2v-1} and the lower sign pertains to Φ_{2v} (S).

The probing signals are then

$$\Phi_1(s) = \sqrt{2\alpha_v} \quad \frac{(-s + \sqrt{\alpha^2 + \beta^2})}{(-s + \alpha)^2 + \beta^2}$$

and

$$\Phi_2(s) = \sqrt{2} \quad \frac{(-s - \sqrt{\alpha^2 + \beta^2}) \quad (-s - \alpha)^2 + \beta^2}{\left[(-s + \alpha)^2 + \beta^2\right]^2}$$

The matrix components for the partial systems are

$$h_{jk} = \frac{1}{2\pi j} \int_c \Phi_j^* \ (-s)\ H_i(s) \ \ \Phi_k(s)\ ds$$

The matrix elements for the partial system $\frac{\partial H}{\partial c_1}$ are found as follows:

$$(h_{11})_{c_1} = \frac{1}{2\pi j} \int_c \Phi_1(-s) \frac{\partial H(s)}{\partial c_1} \ \ \Phi_1(s)\ ds$$

$$= \frac{1}{2\pi j} \int_c \frac{\sqrt{2\alpha} \ (s + \sqrt{\alpha^2 + \beta^2})}{(s + \alpha)^2 + \beta^2} \quad \frac{s}{(s + \alpha)^2 + \beta^2} \quad \frac{\sqrt{2\alpha}(-s + \sqrt{\alpha^2 + \beta^2})}{(-s + \alpha)^2 + \beta^2} \ ds$$

$$= \frac{-2\alpha}{2\pi j} \int_c \frac{s(s^2 - \alpha^2 - \beta^2)\ ds}{\left[(s + \alpha)^2 + \beta^2\right]^2 \left[(s - \alpha)^2 + \beta^2\right]}$$

This integral is evaluated by integrating around the left half-plane, finding the residues in the left half-plane poles. The residue in the second-order pole at $-\alpha - j\beta$ is

$$R_1 = \lim_{s \to -\alpha - j\beta} \quad \frac{d}{ds} \ \frac{s(s^2 - \alpha^2 - \beta^2)}{(s + \alpha - j\beta)^2 \left[(s - \alpha)^2 + \beta^2\right]} \qquad = - \frac{1}{16\alpha^2}$$

The residue R_2, at $-\alpha + j\beta$ is the complex conjugate of R_1:

$$R_2 = -\frac{1}{16\alpha^2}$$

The sum of the residues is $-\dfrac{1}{8\alpha^2}$, and so

$$(h_{11})_{c_1} = (-2\alpha)\left(-\frac{1}{8\alpha^2}\right) = \frac{1}{4\alpha} .$$

$$(h_{12})_{c_1} = \frac{1}{2\pi j} \int_c \Phi_1(-s) \frac{\partial H(s)}{\partial c_1} \Phi_2(s)\, ds$$

$$= \frac{1}{2\pi j} \int_c \frac{\sqrt{2\alpha}\,(s + \sqrt{\alpha^2 + \beta^2})}{(s+\alpha)^2 + \beta^2} \quad \frac{s}{(s+\alpha)^2 + \beta^2}$$

$$\frac{\sqrt{2\alpha}\quad (-s-\alpha)^2 + \beta^2 \quad (-s - \sqrt{\alpha^2 + \beta^2}}{\left[(-s+\alpha)^2 + \beta^2\right]^2} \quad ds$$

$$= \frac{-2\alpha}{2\pi j} \int_c \frac{s\,(s + \sqrt{\alpha^2 + \beta^2})^2 \quad ds}{(s+\alpha)^2 + \beta^2 \quad \left[(s-\alpha)^2 + \beta^2\right]^2}$$

$$= 0$$

$$(h_{21})_{c_1} = \frac{1}{2\pi j} \int_c \Phi_2(-s) \frac{\partial H(s)}{\partial c_o} \Phi_1(s)\, ds$$

$$= \frac{1}{2\pi j} \int_c \frac{\sqrt{2\alpha}\quad (s-\alpha)^2 + \beta^2 \quad (s - \sqrt{\alpha^2 + \beta^2})}{\left[(s+\alpha)^2 + \beta^2\right]^2} \qquad \frac{s}{(s+\alpha)^2 + \beta^2}$$

$$\frac{\sqrt{2\alpha}\quad (-s + \alpha^2 + \beta^2)}{(-s+\alpha)^2 + \beta^2} \quad ds$$

$$= \frac{-2\alpha}{2\pi j} \int_c \frac{s(s^2 - \alpha^2 - \beta^2)\, ds}{\left[(s+\alpha)^2 + \beta^2\right]^3}$$

$$= 0$$

$$(h_{22})_{c_1} = \frac{1}{2\pi j} \int_c \Phi_2(-s) \frac{\partial H(s)}{\partial c_1} \Phi_2(s)\, ds$$

$$= \frac{1}{2\pi j} \int_c \frac{\sqrt{2\alpha}\,(s-\alpha)^2 + \beta^2 \,(s-\sqrt{\alpha^2+\beta^2})}{\left[(s+\alpha)^2+\beta^2\right]^2} \qquad \frac{s}{(s+\alpha)^2+\beta^2}$$

$$\frac{\sqrt{2\alpha}\,(-s-\alpha)^2 + \beta^2}{\left[(-s+\alpha)^2+\beta^2\right]^2} \,(-s-\sqrt{\alpha^2+\beta^2}) \qquad ds$$

$$= \frac{-2\alpha}{2\pi j} \int_c \frac{s(s^2-\alpha^2-\beta^2)\quad ds}{\left[(s+\alpha)^2+\beta^2\right]^2 \quad (s-\alpha)^2+\beta^2}$$

$$= h_{11} = \frac{1}{4\alpha}$$

The matrix elements for $\frac{\partial H}{\partial c_1}$ are found in the same way. The resulting matrices are

$$H_{c_1} = \begin{bmatrix} \frac{1}{4\alpha} & 0 \\ 0 & \frac{1}{4\alpha} \end{bmatrix}$$

and

$$H_{c_0} = \begin{bmatrix} \frac{1}{4(\alpha^2+\beta^2)}, & \frac{1}{4\alpha} \quad \frac{-\alpha+\sqrt{\alpha^2+\beta^2}}{\alpha^2+\beta^2} \\ 0 & \frac{1}{4(\alpha^2+\beta^2)} \end{bmatrix}$$

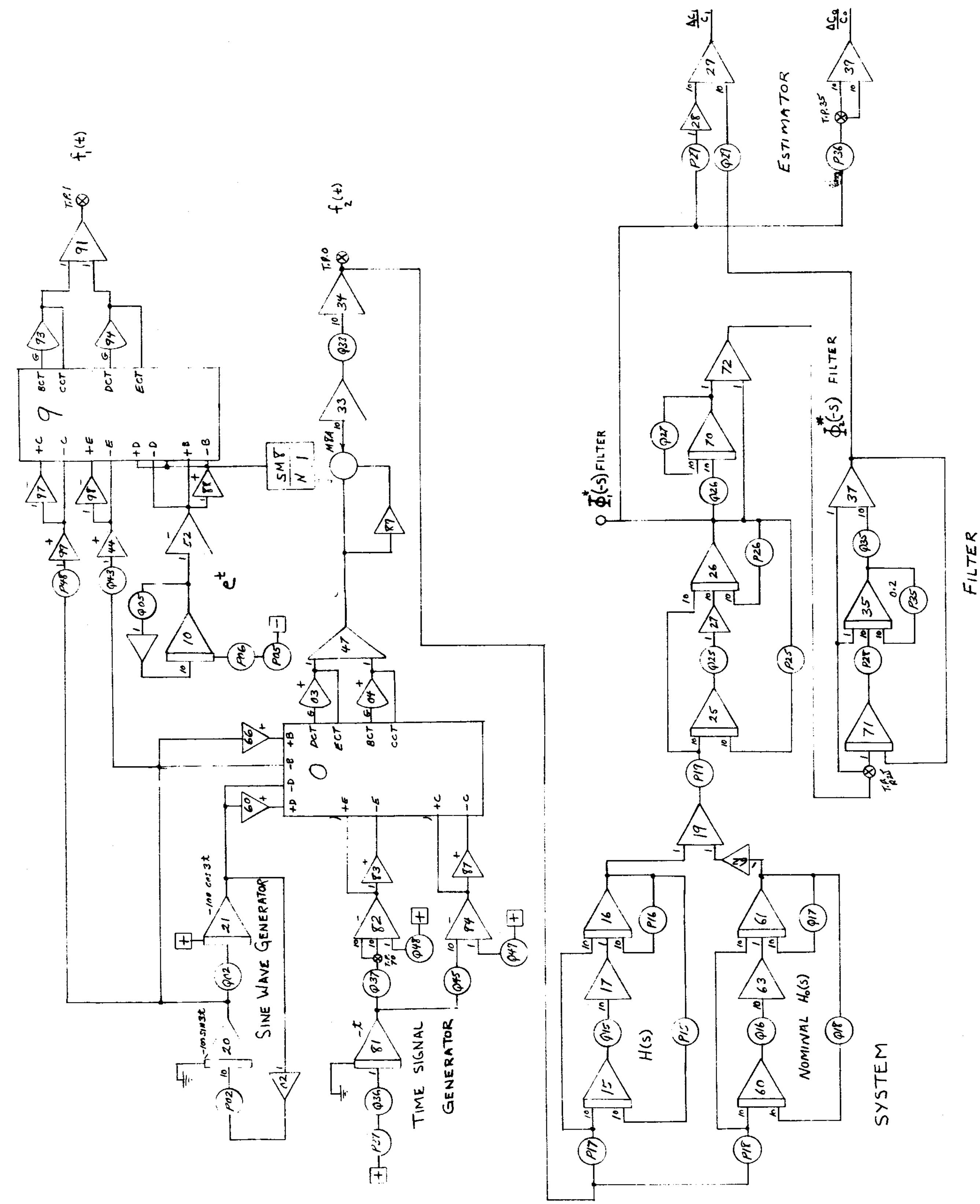


FIGURE 3-17
SECOND ORDER SYSTEM
MEASURING SET-UP

Since there are only two probing signals, the coefficients of $\Phi_1(S)$ and $\Phi_2(S)$ are chosen to be $\cos \Psi$ and $\sin \Psi$ as in the first order example. The calculation of the parameter modulation matrix M is as follows:

$$M = \begin{bmatrix} M_{c_1} & M_{c_o} \end{bmatrix}$$

$$M_{c_1} = \begin{bmatrix} \frac{1}{4\alpha} & 0 \\ 0 & \frac{1}{4\alpha} \end{bmatrix} \begin{bmatrix} \cos \Psi \\ \sin \Psi \end{bmatrix}$$

$$= \begin{bmatrix} \frac{\cos \Psi}{4\alpha} \\ \frac{\sin \Psi}{4\alpha} \end{bmatrix}$$

$$M_{c_o} = \begin{bmatrix} \frac{1}{4(\alpha^2 + \beta^2)}, & \frac{1}{4\alpha} \ \frac{-\alpha + \sqrt{\alpha^2 + \beta^2}}{\alpha^2 + \beta^2} \\ 0 & \frac{1}{4\ (\alpha^2 + \beta^2)} \end{bmatrix} \begin{bmatrix} \cos \Psi \\ \sin \Psi \end{bmatrix}$$

$$= \begin{bmatrix} \frac{\cos \Psi}{4(\alpha^2 + \beta^2)} + \frac{\sin \Psi}{4\alpha} \ \frac{-\alpha \sqrt{\alpha^2 + \beta^2}}{\alpha^2 + \beta^2} \\ \frac{\sin \Psi}{4(\alpha^2 + \beta^2)} \end{bmatrix}$$

Then

$$M = \begin{bmatrix} \frac{\cos \Psi}{4\alpha}, & \frac{\cos \Psi}{4(\alpha^2 + \beta^2)} + \left[\frac{\sin \Psi}{4\alpha}\right] \frac{-\alpha + \sqrt{\alpha^2 + \beta^2}}{\alpha^2 + \beta^2} \\ \\ \frac{\sin \Psi}{4\alpha}, & \frac{\sin \Psi}{4(\alpha^2 + \beta^2)} \end{bmatrix}$$

The maximum value of the determinant of $|\overline{M}M|$ occurs when $\Psi = 90^\circ$. Substitution of $\Psi = 90^\circ$ in the expression for M then yields

$$M = \begin{bmatrix} 0 & \frac{-\alpha + \sqrt{\alpha^2 + \beta^2}}{4\alpha \ (\alpha^2 + \beta^2)} \\ \\ \frac{1}{4\alpha} & \frac{1}{4(\alpha^2 + \beta^2)} \end{bmatrix}$$

In the present example $d_1 = 2$ and $d_o = 10$, so that $\alpha = 1$ and $\beta = 3$.

Then

$$M = \begin{bmatrix} 0 & 0.05406 \\ \\ 0.2500 & 0.02500 \end{bmatrix}$$

$$(M)^{-1} = \begin{matrix} -1.849 & 4.000 \\ 18.49 & 0 \end{matrix}$$

The testing of this second order transfer function was conducted on the analog computer. The main purpose of the test was to establish feasibility of testing second order transfer functions. The probing signal was constructed by time shifting. The transfer function and filters were simulated. The simulated filters were complicated by the complex pole of the transfer function. The probing signal also was more complicated and required more' generation equipment.

The results of the experimentation were good, and we could measure parameter variations in c_o and c_1 independently. The following two figures 3-18 and 3-19 illustrate the data obtained. The parameter variations are for $\pm$ 10%, $\pm$ 20%, $\pm$ 30%. In figure 3-18 the c_o was varied, while c_1 was held at its nominal value. In figure 3-19, c_o was held constant while c_1 was varied. The trace in the middle of the figure is the optimum probing signal.

This concluded the work done on second order transfer functions. The calculation of the estimators for d_o and d_1 was not completed. The calculation of these estimators is even more complicated. We are, however, confident that measurement of these parameters is within our capabilities. We conclude that second order transfer functions can be measured using growing exponential methods.

One other point of interest indicated in both figures 3-18 and

3-19 shows that sampling time for second order systems may not be as critical as first order systems. Both tests clearly show an increased amplitude along the entire functions, and this may possibly be considered in a Go-No-Go Test Scheme.

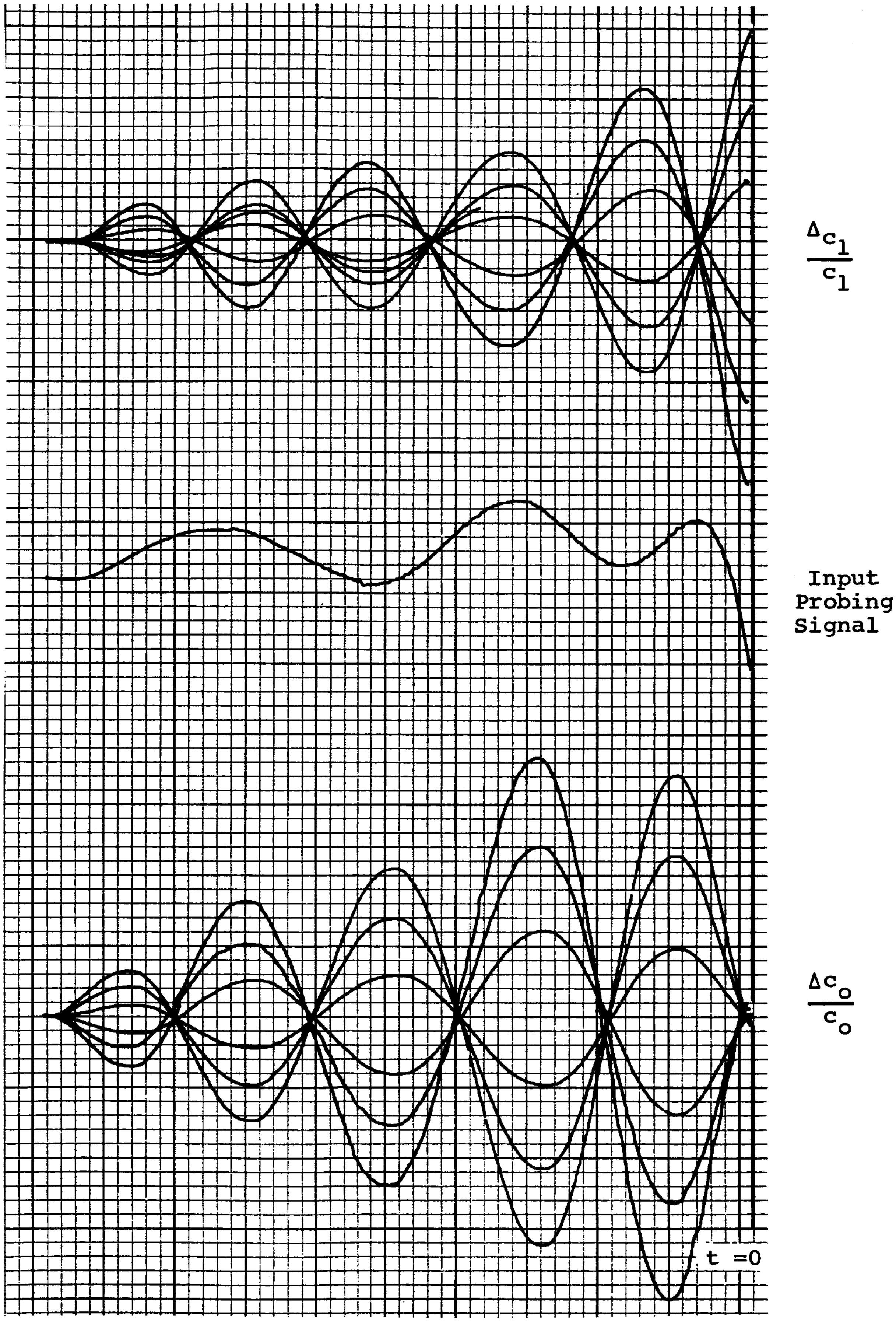


Figure 3-18. Parameter variations of $\frac{\Delta c_o}{c_o}$ with $\frac{\Delta c_1}{c_1} = 0$

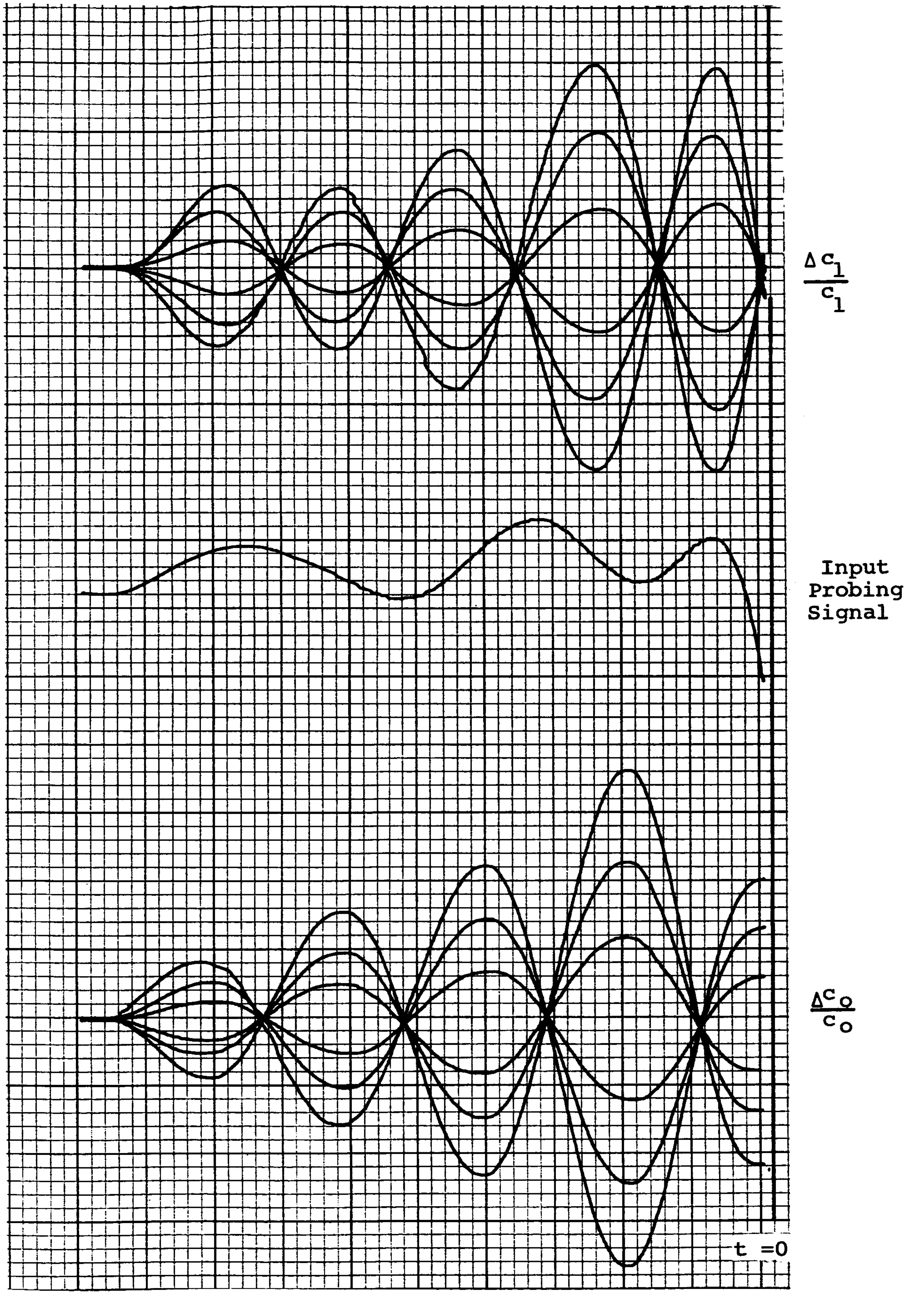


Figure 3-19. Parameter Variations of $\frac{\Delta c_1}{c_1}$ with $\frac{\Delta c_o}{c_o} = 0$

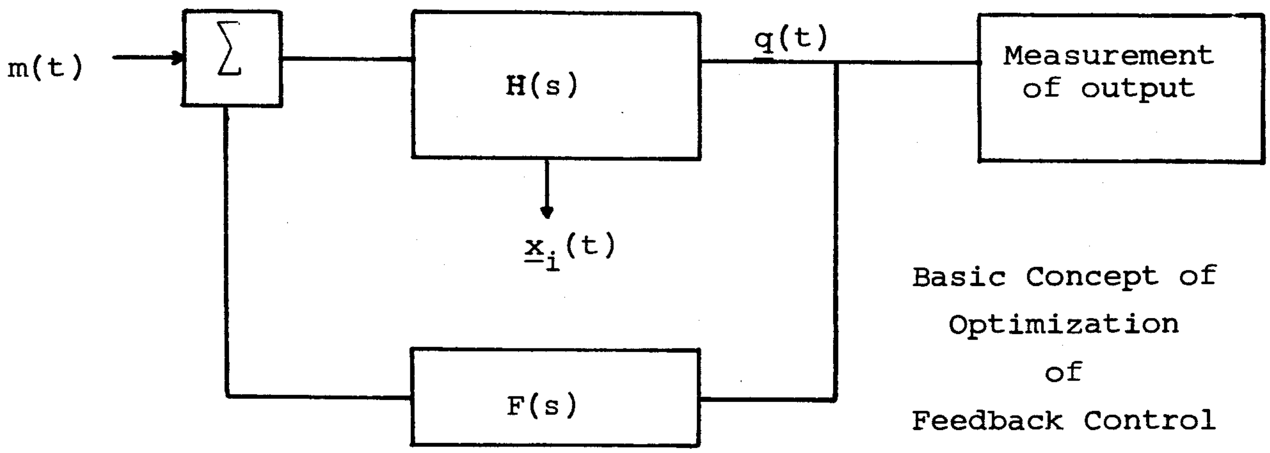
SECTION 4

OPTIMIZATION OF FEEDBACK CONTROL

4.1 INTRODUCTION

The testing of system performance by feedback control has a potential possibility of providing rapid test of operation with relatively small amounts of equipment, when compared to the method of growing exponentials.

The basic concept of optimization of feedback control for system performance testing is illustrated in Figure 4-1.



Basic Concept of Optimization of Feedback Control

Figure 4-1

The transfer function H(s) has a mathematical description called the "dynamic process." The inputs or independent variables of the dynamic process are the "control signals," $m_1(t)$, $m_2(t)$, . . . $m_M(t)$. The outputs, or dependent variables, of dynamic process are called the response signals $q_1(t)$, $q_2(t)$, . . . $q_Q(t)$.

Because the response signals may not be physical variables, a second set of outputs is associated with the dynamic process. These outputs are called the "state signals" $x_1(t)$, $x_2(t)$, . . . $x_N(t)$. State signals require a careful definition, since they are not usually associated with frequency-domain design techniques. For all dynamic processes described by ordinary differential equations, a finite number of variables uniquely determine the distribution of energy or state of the system. The minimal number of signals required to define the state of the dynamic process is equal to the order of the differential equations which describe the system or dynamic process.

With readers who are familiar with analog computer simulations, the outputs of the integrators used to solve a system of differential equations are state signals.

While the mathematical model could be expanded to analyze systems with multiple inputs and multiple outputs, the testing transfer function, as defined in the single parameter study, is limited to one input m(t) and one output q(t). The filtering of the output could result in more outputs as in the growing exponential theory, in this case there is one input and multiple outputs from filtering stages.

A convenient format for writing the description of the dynamic process, or transfer function in terms of the defining state signals is

$$\dot{\underline{x}}(t) = \underline{f}\,(\underline{x}(t),\ \underline{m}(t),\ t).$$

where

$$\underline{x}(t) = \begin{matrix} x_1(t) \\ x_2(t) \\ \cdot \\ \cdot \\ x_N(t) \end{matrix} \qquad \underline{m}(t) = \begin{matrix} m_1(t) \\ m_2(t) \\ \cdot \\ \cdot \\ m_M(t) \end{matrix}$$

which equivalent to the set of first order differential equations.

$$\dot{x}_n(t) = f_n \; [\underline{x}(t), \underline{m}(t), t]$$

$$n = 1, 2, \ldots N$$

The "response equation" is related to the state variables by

$$\underline{q}(t) = \underline{g} \; [\underline{x}(t), t]$$

which is equivalent to the set of equations

$$q_n(t) = g_n \; (\underline{x}(t), t)$$

$$n = 1, 2, \ldots Q$$

where

$$\underline{q}(t) = \begin{matrix} q_1(t) \\ \cdot \\ \cdot \\ \cdot \\ q_Q(t) \end{matrix}$$

In order to classify the use of this notation suppose that the fixed number of the control system is described by the transfer function

$$\frac{q_1(s)}{m_1(s)} = \frac{\sum_{n=o}^{N-1} c_n \, s^n}{\sum_{n=o}^{N} d_n \, s^n}$$

where $q_1(s)$ is the output Laplace transform of $q_1(t)$, $m_1(s)$ is the input Laplace transform of the input $m_1(t)$ and S is the complex frequency variable. By making $d_N = 1$ we lose no generality.

By cross-multiplication the transform is equivalent to

$$\sum_{n=1}^{N} d_n\, q^{(n)}(t) = \sum_{n=o}^{N-1} c_n\, m^{(n)}(t)$$

in the time domain. In addition, a new variable $x_N(t)$ is introduced so that the above equation reduces to

$$\sum_{n=o}^{N} d_n\, (x_n^{(n)}(t)) = m_1(t)$$

$$\sum_{n=o}^{N-1} c_n\, (x_n^{(n)}(t)) = q_1(t)$$

Finally the addition variables are introduced so that

$$\dot{x}_n(t) = x_{n-1}(t) \qquad n = 2, 3, \ldots N$$

and then the equation for $m_1(t)$ reduces to

$$\dot{x}_1(t) = \sum_{n=1}^{N} [-d_{N-n}\; x_n(t)] + m(t)$$

Likewise,

$$q_1(t) = \sum_{n=1}^{N} c_{N-n}\; x_n(t)$$

An analog computer simulation of this transfer function can now be written as

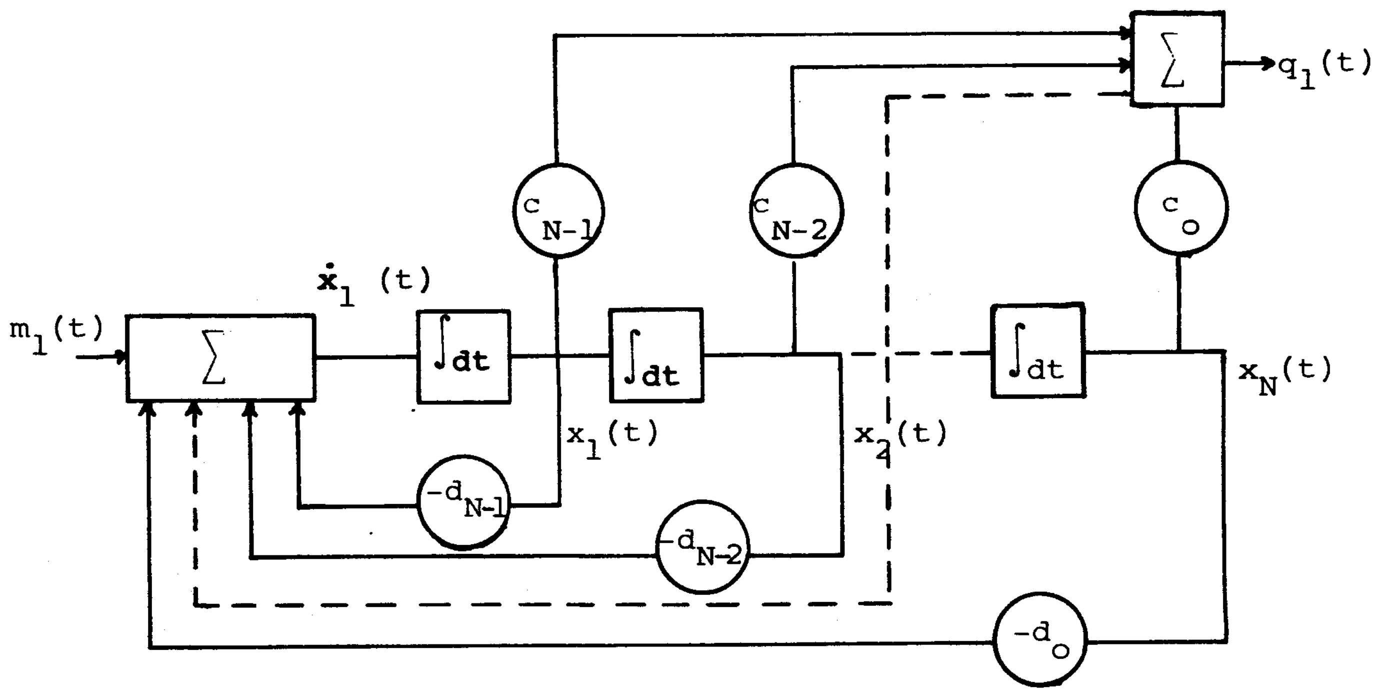


Figure 4-2

Analog Computer Simulation

This simulation represents a generalized transfer function where the c's and the coefficients in the numerator, and the d's are the coefficients in the denominator.

The performance specifications of the system can be assumed (and should be) written in terms of the errors between actual, and desired responses as a matter of choice. Therefore, a control $m_1(t)$ which minimizes or maximizes this criterion is desired.

The instantaneous indications incurred in the system are calculated in terms of some measure called a "performance measure." (Sometimes referred to as an error measure). The performance index is the total performance incurred in the system over the present and future time where the control system operates. This performance index (error index) is found by integrating the performance measure

$$PI = \int_{t}^{T} p(\sigma) \ d\sigma$$

where $p(\sigma)$ the performance measure is

$$p(\sigma) = h \left[\underline{q}(t), \underline{m}(t), t)\right]$$

In the single parameter testing problem, this theory has an application in possibly two ways. The first is to form a performance measure which gives an indication of the change in parameter or parameters. The second is to form a performance measure which will give an indication of the sensitivity of the output to a change in a parameter or parameters. Other performance measurements may also be possible.

4.2 PARAMETER DEVIATION MEASURE

The performance measure for the system could be written in terms of parameter changes by defining an additional set of state variables. One state for each parameter

$$
\begin{array}{llllll}
x(t)_{N+1} & = & c_o & = & \alpha_o \\
x(t)_{N+2} & = & c_1 & = & \alpha_1 \\
\quad\vdots & & & & \\
x(t)_{2N+1} & = & c_{N-1} & = & \alpha_{N-1} \\
x(t)_{2N} & = & d_o & = & \alpha_N \\
x(t)_{2N+1} & = & d_1 & = & \alpha_{2N+1} \\
\vdots & & \vdots & & \\
x(t)_{3N-1} & & d_N & = & \alpha_{2N-1}
\end{array}
$$

where $\dot{x}_n(t) = 0$ for $N + 1 \leq n \leq 3N - 1$

The performance measure could be

$$p(t) = \sum_{n = N+1}^{n = 3N+1} (x_n(t) - \alpha_n) \ \phi_n(t)$$

where the $\phi_1\ (\alpha)$, $\phi_2\ (\sigma)$, . . . $\phi_n\ (\sigma)$ are weighting factors, when $t \leq \sigma \leq T$ and T is some future time.

There is a great deal of information on the solution of optimal control for this type of performance measure. The "quadratic" form, created by squaring the difference between the measured parameter $x_n(t)$ and the desired parameter α_n has been studied by many engineers, and scientist. Dynamic Programming, Parametric Expansion, and Calculus of variations are all methods applied to the solution of this problem.[18]

4.3 A SENSITIVITY MEASURE

The performance measure could also be formed by a combination of sensitivities of the output $q_1(t)$ to the parameter considered. As in the growing exponential case, the partial systems were probed by an input signal and measurements made on the output. These partial systems are sensitivity measures. Formulating a performance measure in terms of the sum of the sensitivities is the same as investigating the change in the first order terms of the Taylor series when the particular systems are normalized.

The definition of sensitivity to be used is

$$\frac{\alpha \; d\, q(t)}{q(t) \; d\, \alpha} = S_{\alpha}^{q}$$

The performance measure would then become

$$\sum_{n-1}^{2N-1} S_{\alpha_n}^{q(t)} = p(t)$$

4.4 ILLUSTRATIVE EXAMPLE

For illustration of the possibilities of optimum feedback control, the following example will use, as a performance measure, the summation of the sensitivities with respect to the parameters.

Let the transfer function to be tested be

$$H(s) = \frac{K/\tau}{S + 1/\tau} = \frac{c_o}{d_1 \, S + d_o}$$

The analog simulation of this transfer function is illustrated in Figure 4-3.

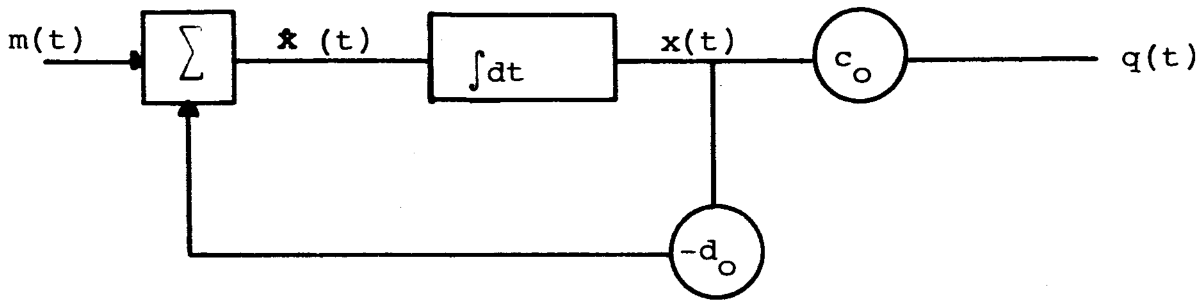


Figure 4-3

Analog Simulation of Transfer Function

The state equations describing this transfer function are

$$\dot{x}(t) = -x(t)\, d_o + m(t)$$

$$q(t) = c_o\, x(t) = \frac{c_o}{d_o}\left\{-\dot{x}(t) + m(t)\right\}$$

The sensitivity of q with the parameters c_o and d_o are,

$$S^{q}_{d_o} = \frac{d_o}{c_o\, x_1(t)}\left\{-\dot{x}(t) + m(t)\right\}$$

$$S^{q}_{c_o} = \frac{c_o}{c_o\, x_1(t)}\left\{-\dot{x}(t) + m(t)\right\}$$

The performance measure is

$$p(t) = S^{q}_{c_o} + S^{q}_{d_o} = \left\{\frac{d_o}{c_o} + 1\right\}\left\{-\frac{m(t) - \dot{x}(t)}{x(t)}\right\}$$

The performance index is therefore;

$$\int_{t}^{\tau} \left[S_{d_o}^{q} + S_{c_o}^{q} \right] dt = \int_{o}^{\tau} \left[S_{d_o}^{q} + S_{c_o}^{q} \right] dt$$

where t, present time, is assumed to be zero.

This problem is a particular class of optimization problem. The solution is of the singular form, as solved by Johnson and Gibson.[19] Using the Johnson and Gibson method of solution, the Hamiltonian function is formed

$$H = \sum p_i \dot{x}_i \quad (i = o, 1, 2, \ldots n)$$

where

p_i = Lagrange multiplier function

$\dot{x}_i$ = The system equation

n = Number of system equation, and

$\dot{x}_o(t)$ = is the performance measure

$x_o(\tau)$ = performance index

$x_o(o)$ = 0

Therefore, the Hamiltonian function becomes

$$H = p_1 \left\{ m(t) - x(t)\, d_o) \right\} + p_o \left[\frac{d_o}{c_o} + 1 \right] \left\{ \frac{m(t) - \dot{x}(t)}{x(t)} \right\}$$

The ajoint equations are

$$-\dot{p}_1 = \frac{\partial H}{\partial x(t)} = -p_1 d_o - \frac{p_o \left[\frac{d_o}{c_o} + 1 \right] \{ m(t) - \dot{x}(t) \}}{(x(t))^2}$$

$$-\dot{p}_o = 0 \quad \therefore \quad p_o = \text{constant} = 1$$

and the Hamiltonian function can then be written as

$$H = \left[-p_1\, x(t)\, d_o - \left[\frac{d_o}{c_o} + 1\right] \frac{\dot{x}(t)}{X(t)}\right] + m(t)\left[p_1 + \left\{\frac{d_o}{c_o} + 1\right\} \frac{1}{x(t)}\right]$$

$$H = I + m(t)\ F$$

The conditions for a maximum can be found by setting

$$\dot{I} = \dot{I} = \ddot{I} \ldots = 0$$

$$\dot{F} = \dot{F} = \ddot{F} \ldots = 0$$

where

$$I = -p_1\, x(t)\ d_o - \left[\frac{d_o + c_o}{c_o}\right] \frac{\dot{x}(t)}{x(t)}$$

$$F = p_1 + \left[\frac{d_o + c_o}{c_o}\right] \frac{1}{x(t)}$$

I = 0 implies

$$p_1\ d_o\ \ x(t) = \left[-\frac{d_o + c_o}{c_o}\right] \frac{\dot{x}(t)}{x(t)}$$

F = 0 implies

$$p_1 = \left[-\frac{d_o + c_o}{c_o}\right] \frac{1}{x(t)}$$

Therefore,

$$x(t) = \frac{1}{d_o}\ \dot{x}(t)$$

The solution for the control variable can now be obtained from the system equation

$$\dot{x}(t) = -x(t)\, d_o + m(t)$$

$$\dot{x}(t) = -\dot{x}(t) + m(t)$$

$$m(t) = +2\dot{x}(t) = +2\, d_o\, x(t)$$

The solution of the equation

$$x_1(t) = \dot{x}(t)/d_o$$

can be determined using Laplace transforms

$$d_o\, x(s) - S\, x(s) - x(o) = 0$$

$$x(s) = \frac{+\, x(o)}{S - d_o}$$

$$x(t) = +\, x(o)\, e^{+ d_o t}$$

therefore, $$m(t) = +\, 2x(o)\, d_o\, e^{+ d_o t}$$

Thus, the optimum input signal is a growing exponential. The testing of the transfer function is accomplished as in Figure 4-4.

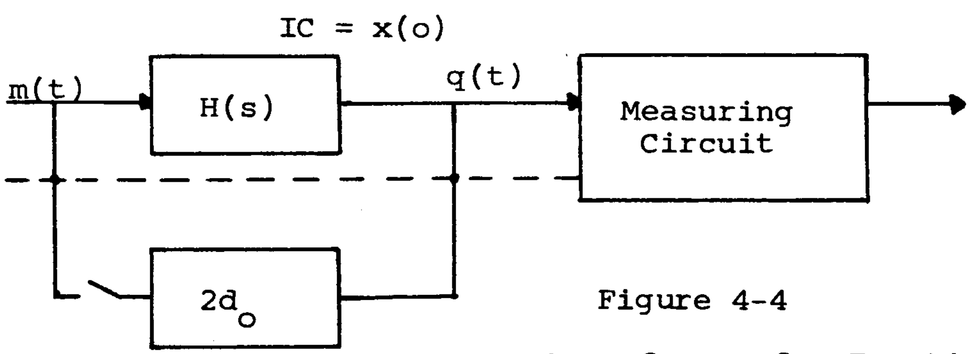


Figure 4-4

Testing of Transfer Function

x(o) is the necessary initial condition of x(t) at t = 0, and the feedback is positive, therefore, causing a growing exponential to be the output and input.

Applying portions of the theory of growing exponentials, the output can be filtered and sampled at the proper instant so that the sample value is unity.

Let $q(t) = c_o \, x(-a) \, e^{+ d_o (t)}$ be defined for negative time, $-a < t < 0$. Then the filter which matches this is

$$\Phi_q^*(-s) = \frac{1/c_o}{S + d_o}$$

Sampling at time $t = 0$ is equivalent to

$$\int_{-\infty}^{+\infty} q^*(t) \; h(t) \; dt \approx 1$$

where $h(t)$ is the impulse reponse of

$$\Phi_q^*(-s)$$

With the filter and sampling, a measure of the total system performance will be established. If the output of the filter is different from unity by a given percentage, the circuit could be classified--failed.

The extension of the procedure to higher order systems is desirable to investigate feasibility and limitations.

SECTION 5

CONCLUSION AND RECOMMENDATIONS

1. It has been proven that the Taylor series expansion is valid and can be used for measuring system changes.
2. In the growing exponential method, the mathematics involved in obtaining an estimate of the parameter changes is lengthy, and has been the biggest problem.
3. It has been proven that measurements can be taken with noisy parameters and noisy signals.
4. Knowledge and understanding of the basic measuring process has been obtained.
5. Absolute measurement accuracy is a function of system complexity, as illustrated by the result of the two first order transfer functions examined.
6. For the particular problem examined, good results were obtained using the optimum feedback control method.

RECOMMENDATIONS

The growing exponential method needs to be expanded to higher than second order systems.

The feedback control method needs more investigation to determine the feasibility and limitations of the approach.

Active networks need to be investigated so that techniques will be available in Phase III for implementation.

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